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# SCALING RULES FOR UNEQUAL BEAM-BEAM PARAMETERS


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ABSTRACT

We derive the scaling rules that yield the spot size at the interaction point (IP) and the beam currents as the nominal beam-beam parameters move away from their nominally-specified values. These rules allow reaching new specifications in a very simple way should one want to modify the design values of the beam-beam parameters. The scaling rules are derived under four basic constraints: (1) the beta functions at the IP are fixed; (2) the rms spot sizes at the IP are pairwise equal; (3) the beam energies are fixed, and (4) the nominal luminosity and collision frequency are fixed. In addition, we impose the constraint that the beam-beam parameters must either obey transparency symmetry or *x-y* symmetry.


## 1. Introduction.

The fact that PEP-II (like all other "factory"-type colliders presently considered) has two separate and quite different rings, implies that the machine will almost certainly have to be optimized such that most, perhaps all, the beam parameters will be different in the two rings. From the beam-beam perspective, the Conceptual Design Report[1] (CDR) and most multiparticle simulation studies so far[2] have assumed that all four beam-beam parameters are equal. This equality is the simplest case of a transparency symmetry[3] that is intended to balance the beams in order to compensate for their unequal energies. The constraints imposed by this symmetry have the additional practical benefit of reducing the number of different parameters. However, the symmetry is only approximately valid even at the nominal design level[1] and, moreover, it is further broken by the dynamics, particularly from the effects of the parasitic collisions. It is natural, therefore, to explore the optimization with respect to the beam-beam parameters by exploring the dynamical behavior upon departure from the full equality mentioned above. In fact, a first step in this direction has been taken[4] in which multiparticle simulations showed that, under certain conditions, the beam-beam dynamics does prefer unequal beam-beam parameters.

In this note we present simple scaling rules that yield the nominal beam sizes and beam currents as the beam-beam parameters vary away from their nominally-specified values. These scaling rules are derived under four basic constraints: (1) the beta functions at the IP are fixed and their vertical-to-horizontal ratios are equal in the two beams; (2) the rms spot sizes at the IP are pairwise equal; (3) the beam energies are fixed; and (4) the nominal luminosity and collision frequency are fixed. In addition, for simplicity, we require that the beam-beam parameters must either obey transparency symmetry or *x*-*y* symmetry (see below). We ignore all dynamical effects from the beam-beam interaction, including all effects from the parasitic collisions.

In beam-beam studies the word "nominal" is usually used to mean "in the absence of beam-beam effects." Thus nominal quantities, such as the beam sizes, the emittances or the luminosity, carry a subscript "0" to distinguish them from the corresponding "dynamical" quantities, which do take into account the effects of the beam-beam collisions, and which are written with no subscript. As mentioned in the previous paragraph, in this note we present scaling rules for the nominal, not the dynamical, quantities. These nominal quantities are scaled away from a reference set of values which we denote with an overbar. In the applications to PEP-II, we assume that the reference values are those in the CDR, summarized in Table 1 below.

## 2. Constraints and basic formulas.

Concretely, the constraints are:

- The nominal rms beam sizes at the IP are pairwise equal, namely

$$\sigma_{0x,+} = \sigma_{0x,-} \equiv \sigma_{0x}\,, \quad \sigma_{0y,+} = \sigma_{0y,-} \equiv \sigma_{0y} \tag{1}$$

although their actual values can vary away from the reference values.

- The beta functions at the IP are fixed and satisfy the condition

$$\left(\frac{\beta_y}{\beta_x}\right)_+ = \left(\frac{\beta_y}{\beta_x}\right)_- \equiv r_\beta \tag{2}$$

- The nominal luminosity $\mathscr{L}_0$ and the collision frequency $f_c$ are also fixed, and the luminosity formula is assumed to correspond to Gaussian bunches, namely

$$\mathscr{L}_0 = f_c \frac{N_+ N_-}{\sigma_{0x}\sigma_{0y}} \tag{3}$$

- The energies of the two beams are fixed,

$$E_+, E_- = \text{ fixed} \tag{4}$$

Under these constraints, the only primary parameters that can vary under the scaling studied here are the numbers of particles per bunch, $N_\pm$ (or, equivalently, the beam currents), and the nominal rms beam sizes at the IP, $\sigma_{0x}$ and $\sigma_{0y}$.

Constraint (1) obviously implies that there is a single (rather than two) beam aspect ratio at the IP, namely

$$\left(\frac{\sigma_{0y}}{\sigma_{0x}}\right)_+ = \left(\frac{\sigma_{0y}}{\sigma_{0x}}\right)_- \equiv r \tag{5}$$

When this equation is combined with constraint (2), it also implies that there is a single nominal-emittance ratio,

$$\left(\frac{\varepsilon_{0y}}{\varepsilon_{0x}}\right)_+ = \left(\frac{\varepsilon_{0y}}{\varepsilon_{0x}}\right)_- \equiv r_\varepsilon \tag{6}$$

where we have used the obvious relation

$$r = \sqrt{r_\beta r_\varepsilon} \tag{7}$$

We also assume that the expressions for the beam-beam parameters correspond to that of Gaussian bunches. Thus the vertical beam-beam parameter of a positron at the center of the bunch is given by

$$\xi_{0y,+} = \frac{r_0\, N_- \beta_{y+}}{2\pi\gamma_+ \sigma_{0y,-}(\sigma_{0x,-} + \sigma_{0y,-})} \tag{8}$$

where $r_0$ is the classical electron radius and $\gamma_+$ is the usual relativistic factor of the positron. The corresponding expressions for the horizontal beam-beam parameter of the positron, and those for the electron, are obtained from the above by the exchanges $x \leftrightarrow y$ and/or $+ \leftrightarrow -$.

### 3. Scaling under transparency symmetry.

For our purposes, "transparency symmetry" means here that, in addition to constraints (1–4), the nominal beam-beam parameters obey

$$\xi_{0x,+} = \xi_{0x,-} \equiv \xi_{0x}, \quad \xi_{0y,+} = \xi_{0y,-} \equiv \xi_{0y} \tag{9}$$

with $\xi_{0x}$ in general different from $\xi_{0y}$. In this case the beam-beam parameters and the ratios $r$, $r_\varepsilon$ and $r_\beta$ are related by[5]

$$\frac{\xi_{0y}}{\xi_{0x}} = \sqrt{\frac{r_\beta}{r_\varepsilon}} = \frac{r_\beta}{r} \tag{10}$$

and the nominal luminosity, Eq. (3), has the simplified expression[5]

$$\mathscr{L}_0 = K(1+r)\xi_{0y}\left(\frac{EI}{\beta_y}\right)_{+,-} \tag{11}$$

where $K$ is a constant and the subscript $\pm$ means that the quantity in parenthesis can be computed from either beam.

In order to derive the scaling expression for the beam currents $I_\pm$ we first note that constraints (2) and (4) and Eq. (8) imply

$$I_\pm \propto N_\pm \propto \frac{1}{(1+r)\xi_{0y}} = \frac{1}{\xi_{0y} + r_\beta\, \xi_{0x}} \tag{12}$$

Now this proportionality must be valid, in particular, for the reference quantities since these satisfy all the relevant constraints. Thus one can immediately write the scaling rule for $I_\pm$,

$$I_\pm = \bar{I}_\pm \times F(\xi_{0x}, \xi_{0y}) \tag{13}$$

where $F$ is the scaling function

$$F(\xi_{0x}, \xi_{0y}) \equiv \frac{\bar{\xi}_{0y} + r_\beta\, \bar{\xi}_{0x}}{\xi_{0y} + r_\beta\, \xi_{0x}} \tag{14}$$

A scaling rule for the $\sigma_0$'s is derived by noting that Eqs. (3) and (10) imply

$$\sigma_{0x}\sigma_{0y} \propto N_+ N_- \qquad \text{and} \qquad \frac{\sigma_{0y}}{\sigma_{0x}} \propto \frac{\xi_{0x}}{\xi_{0y}} \tag{15}$$

thus one finds

$$\begin{aligned} \sigma_{0x} &= \bar{\sigma}_{0x} \times \sqrt{\frac{\xi_{0y}\,\bar{\xi}_{0x}}{\xi_{0x}\,\bar{\xi}_{0y}}\, F(\xi_{0x}, \xi_{0y})} \\ \sigma_{0y} &= \bar{\sigma}_{0y} \times \sqrt{\frac{\xi_{0x}\,\bar{\xi}_{0y}}{\xi_{0y}\,\bar{\xi}_{0x}}\, F(\xi_{0x}, \xi_{0y})} \end{aligned} \tag{16}$$

It should be pointed out that, in this kind of scaling, the beam aspect ratio does not remain constant but scales as

$$r = \bar{r} \times \frac{\xi_{0x}\,\bar{\xi}_{0y}}{\xi_{0y}\,\bar{\xi}_{0x}} \tag{17}$$

## 4. Scaling under horizontal-vertical symmetry.

By “horizontal-vertical symmetry,” or “$x$-$y$ symmetry,” we mean that, in addition to the primary constraints (1–4), the beam-beam parameters satisfy

$$\xi_{0x,+} = \xi_{0y,+} \equiv \xi_{0+}\,, \quad \xi_{0x,-} = \xi_{0y,-} \equiv \xi_{0-} \tag{18}$$

with $\xi_{0+}$ in general different from $\xi_{0-}$. In this case all three ratios $r$ , $r_\varepsilon$ and $r_\beta$ are equal,[5]

$$r = r_\beta = r_\varepsilon \tag{19}$$

and the expression for the nominal luminosity simplifies to

$$\mathscr{L}_0 = K(1+r)\left(\xi_0 \frac{EI}{\beta_y}\right)_{+,-} \tag{20}$$

In this last equation all quantities except $\xi_0$ and $I$ are fixed on account of our assumed constraints and Eq. (19). Thus we immediately find

$$I_\pm \propto \frac{1}{\xi_{0\pm}} \tag{21}$$

and so the scaling rule for $I_\pm$ is

$$I_\pm = \bar{I}_\pm \times \frac{\bar{\xi}_{0\pm}}{\xi_{0\pm}} \tag{22}$$

Since the aspect ratio $r$ is a scaling constant in this case, the luminosity formula (3), in combination with Eq. (5), implies

$$\sigma_{0x} \propto \sqrt{N_+ N_-} \quad \text{and} \quad \sigma_{0y} \propto \sqrt{N_+ N_-} \tag{23}$$

which, when combined with Eq. (22), yields the scaling rule for the $\sigma_0$’s,

$$\begin{aligned} \sigma_{0x} &= \bar{\sigma}_{0x} \times \sqrt{\frac{\bar{\xi}_{0+}\,\bar{\xi}_{0-}}{\xi_{0+}\,\xi_{0-}}} \\ \sigma_{0y} &= \bar{\sigma}_{0y} \times \sqrt{\frac{\bar{\xi}_{0+}\,\bar{\xi}_{0-}}{\xi_{0+}\,\xi_{0-}}} \end{aligned} \tag{24}$$

## 5. Applications to PEP-II.

Eqs. (13), (16), (22) and (24) constitute the basic results of this note. In both kinds of scaling there are two independent scaling variables, namely $\xi_{0x}$ and $\xi_{0y}$ in the first case, or $\xi_{0+}$ and $\xi_{0-}$ in the second. For the purposes of numerical applications, we fix $\xi_{0x}$ or $\xi_{0-}$ (depending on which symmetry we adopt) and vary $\xi_{0y}$ or $\xi_{0+}$, respectively.

In numerical applications to PEP-II we assume that the reference values for the bunch currents, particles per bunch and rms beam sizes are given by Table 1, which is taken from the CDR.[1] We emphasize that the luminosity, collision frequency and beta functions at the IP are fixed at the values in Table 1.

***Table 1. Reference values for PEP-II primary parameters (from the CDR).***

| | LER ($e^+$) | HER ($e^-$) |
|---|---|---|
| $\mathcal{L}_0$ [$cm^{-2}$ $s^{-1}$] | $3 \times 10^{33}$ | |
| $f_c$ [MHz] | 238.000 | |
| $E$ [GeV] | 3.1 | 9.0 |
| $\overline{N}$ | $5.630 \times 10^{10}$ | $2.586 \times 10^{10}$ |
| $\bar{I}$ [A] | 2.147 | 0.986 |
| $\bar{\varepsilon}_{0x}$ [nm – rad] | 61.27 | 45.95 |
| $\bar{\varepsilon}_{0y}$ [nm – rad] | 2.451 | 1.838 |
| $\beta_x$ [m] | 0.375 | 0.500 |
| $\beta_y$ [m] | 0.015 | 0.020 |
| $\bar{\sigma}_{0x}$ [$\mu$m] | 151.6 | 151.6 |
| $\bar{\sigma}_{0y}$ [$\mu$m] | 6.063 | 6.063 |
| $\bar{\xi}_{0x}$ | 0.03 | 0.03 |
| $\bar{\xi}_{0y}$ | 0.03 | 0.03 |

Figs. 1 and 2 show the spot sizes at the IP plotted vs. $\xi_{0y}$ for fixed values of $\xi_{0x}$ in the case of scaling under transparency symmetry, Eq. (16). Fig. 3 shows the corresponding beam currents. In all case we show with a cross the reference values corresponding to Table 1.

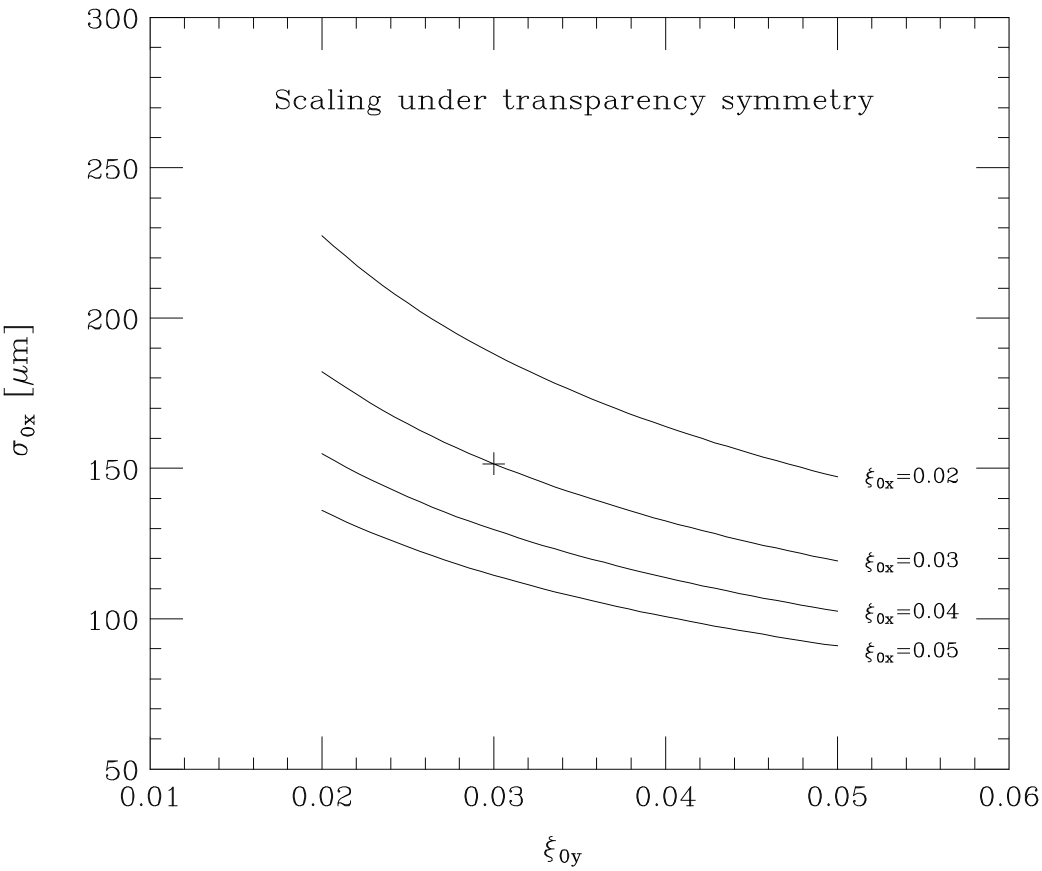


***Fig. 1. Scaling of the horizontal spot size at the IP in the case of transparency symmetry. The cross represents the reference (CDR) value, listed in Table 1.***

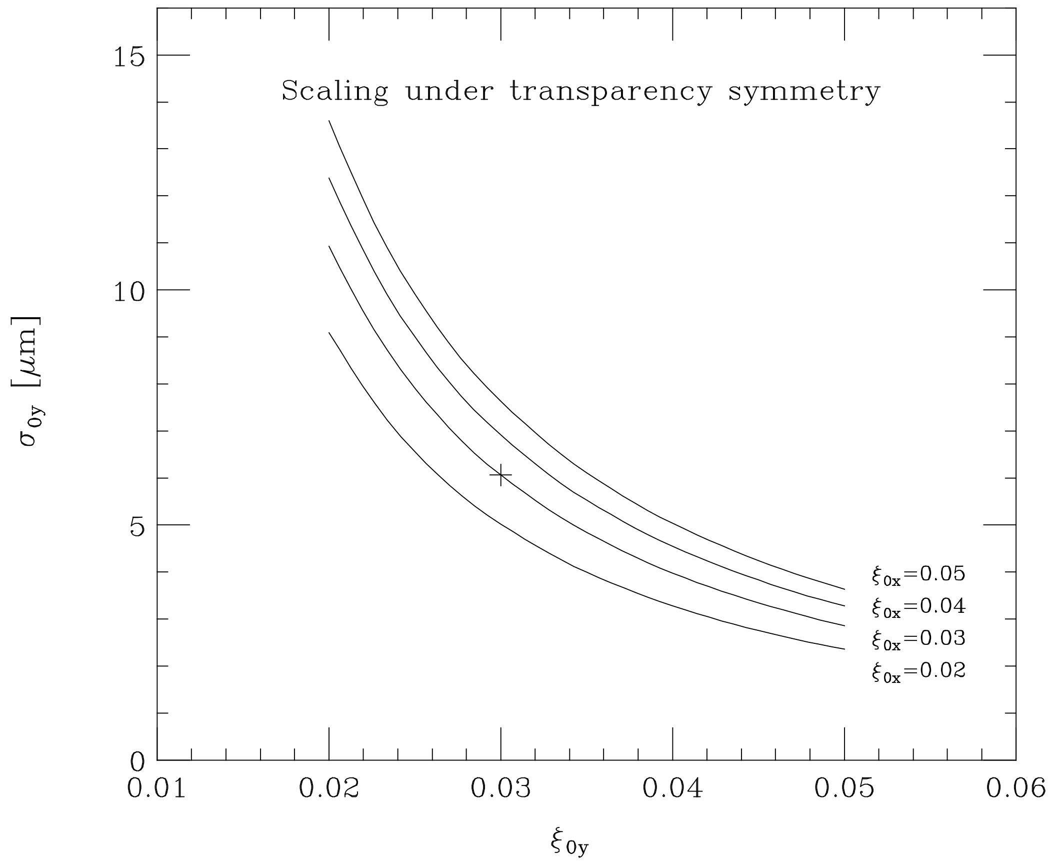


***Fig. 2. Scaling of the vertical spot size at the IP in the case of transparency symmetry. The cross represents the reference (CDR) value, listed in Table 1.***

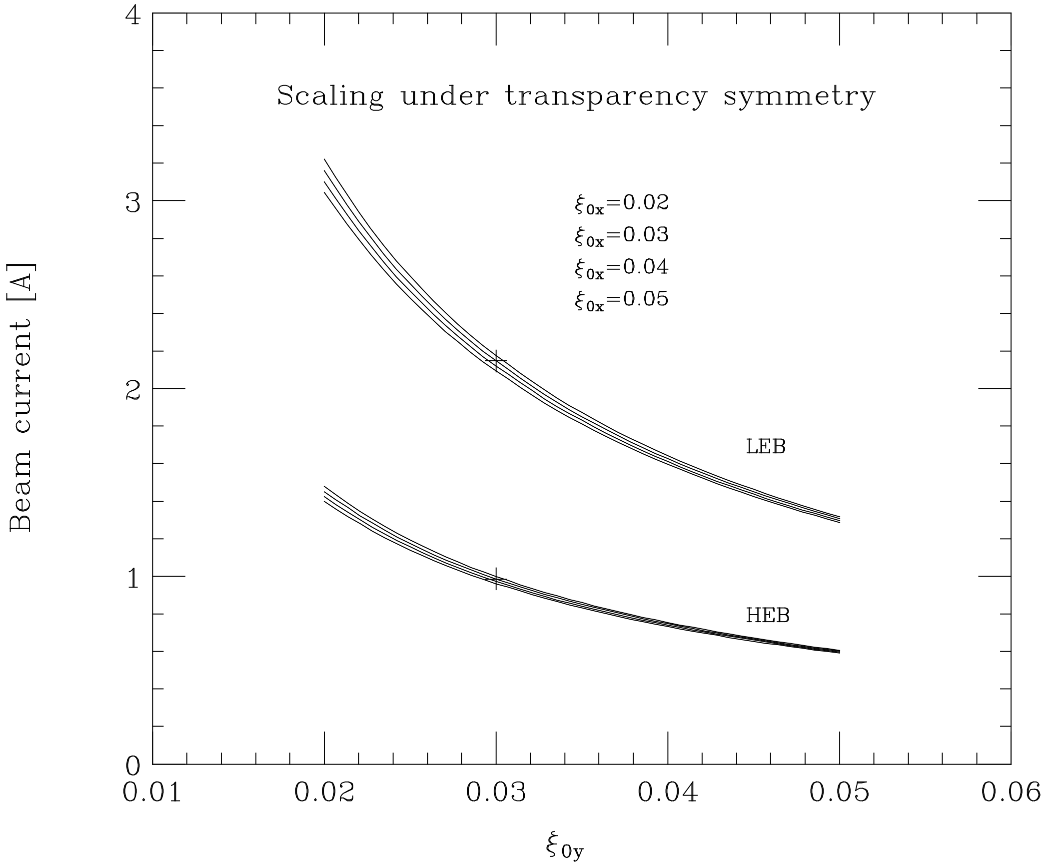


***Fig. 3. Scaling of the beam currents in the case of transparency symmetry. The cross represents the reference (CDR) values, listed in Table 1. The four curves in each case correspond to the four values of $\xi_{0x}$ in the order shown.***

It should be noted that, for fixed $\xi_{0y}$, the horizontal spot size $\sigma_{0x}$ varies inversely with $\xi_{0x}$ while $\sigma_{0y}$ varies directly with $\xi_{0x}$. This can be understood as follows: since $r_\beta$ is small ($r_\beta$ =0.04), we can approximate Eq. (14) to 0-th order in $r_\beta$ by

$$F(\xi_{0x},\xi_{0y}) \approx \frac{\overline{\xi}_0}{\xi_{0y}} = \frac{0.03}{\xi_{0y}} \tag{25}$$

so that the rms spot sizes and beam currents scale approximately as

$$
\begin{aligned}
\sigma_{0x} &\approx \bar{\sigma}_{0x} \times \frac{0.03}{\sqrt{\xi_{0x}\xi_{0y}}} \\
\sigma_{0y} &\approx \bar{\sigma}_{0y} \times 0.03 \times \sqrt{\frac{\xi_{0x}}{\xi_{0y}^3}} \\
I_{\pm} &\approx \bar{I}_{\pm} \times \frac{0.03}{\xi_{0y}}
\end{aligned}
\tag{26}
$$

This last equation shows that, to this order of approximation, the beam currents do not depend on $\xi_{0x}$. The dependence on $\xi_{0x}$ is of first order in $r_\beta$ and hence weak, as seen in Fig. 3.

Figs. 4, 5 and 6 show the spot sizes and currents for the case of scaling under *x*-*y* symmetry. In this case the *exact* scaling rules follow from Eqs. (22) and (24),

$$
\begin{aligned}
\sigma_{0x} &= \bar{\sigma}_{0x} \times \frac{0.03}{\sqrt{\xi_{0+}\xi_{0-}}} \\
\sigma_{0y} &= \bar{\sigma}_{0y} \times \frac{0.03}{\sqrt{\xi_{0+}\xi_{0-}}} \\
I_{\pm} &= \bar{I}_{\pm} \times \frac{0.03}{\xi_{0\pm}}
\end{aligned}
\tag{27}
$$

These expressions show that $\sigma_{0x}$ and $\sigma_{0y}$ scale together, that the LEB current scales only with $\xi_{0+}$, and that the HEB current scales only with $\xi_{0-}$.

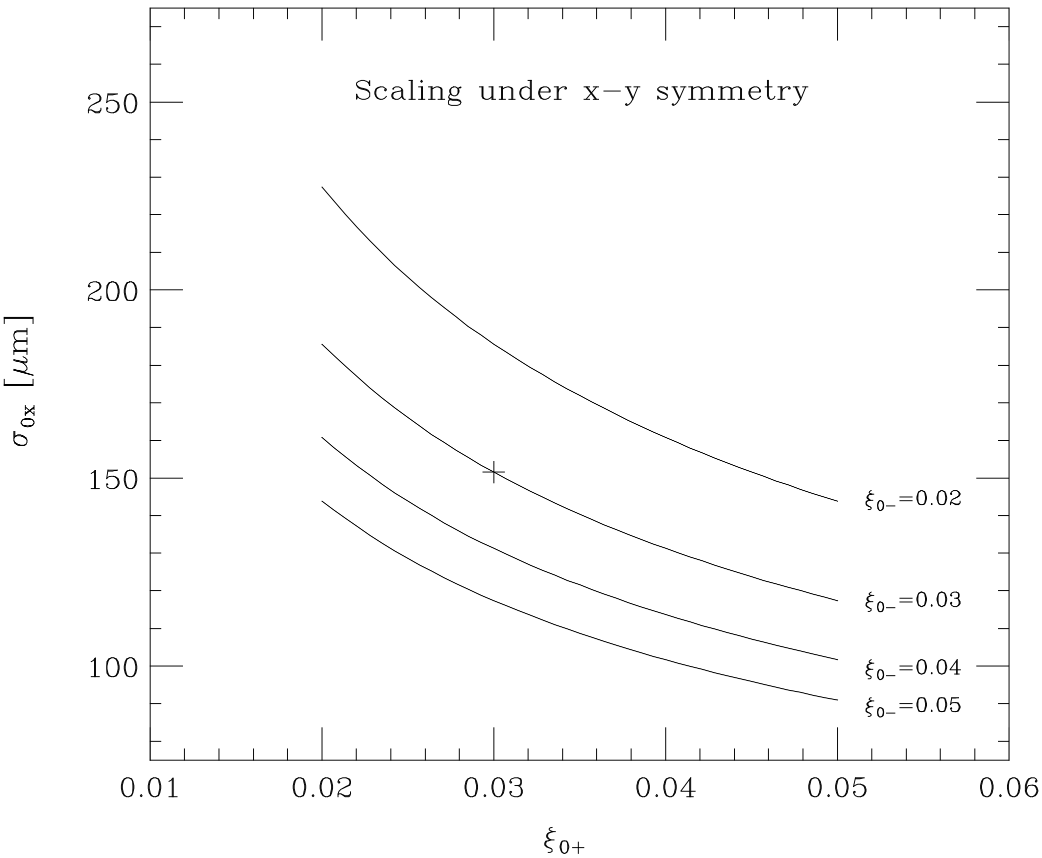


***Fig. 4. Scaling of the horizontal spot size at the IP in the case of x-y symmetry. The cross represents the reference (CDR) value, listed in Table 1.***

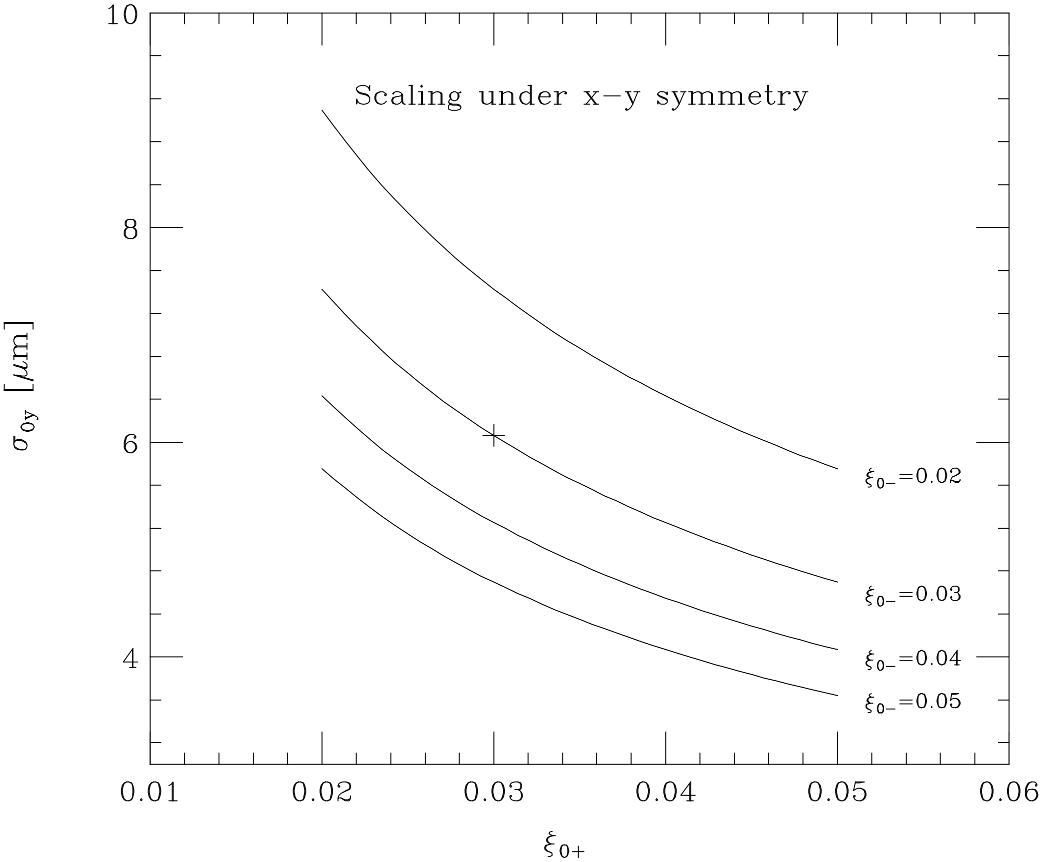


***Fig. 5. Scaling of the vertical spot size at the IP in the case of x-y symmetry. The cross represents the reference (CDR) value, listed in Table 1.***

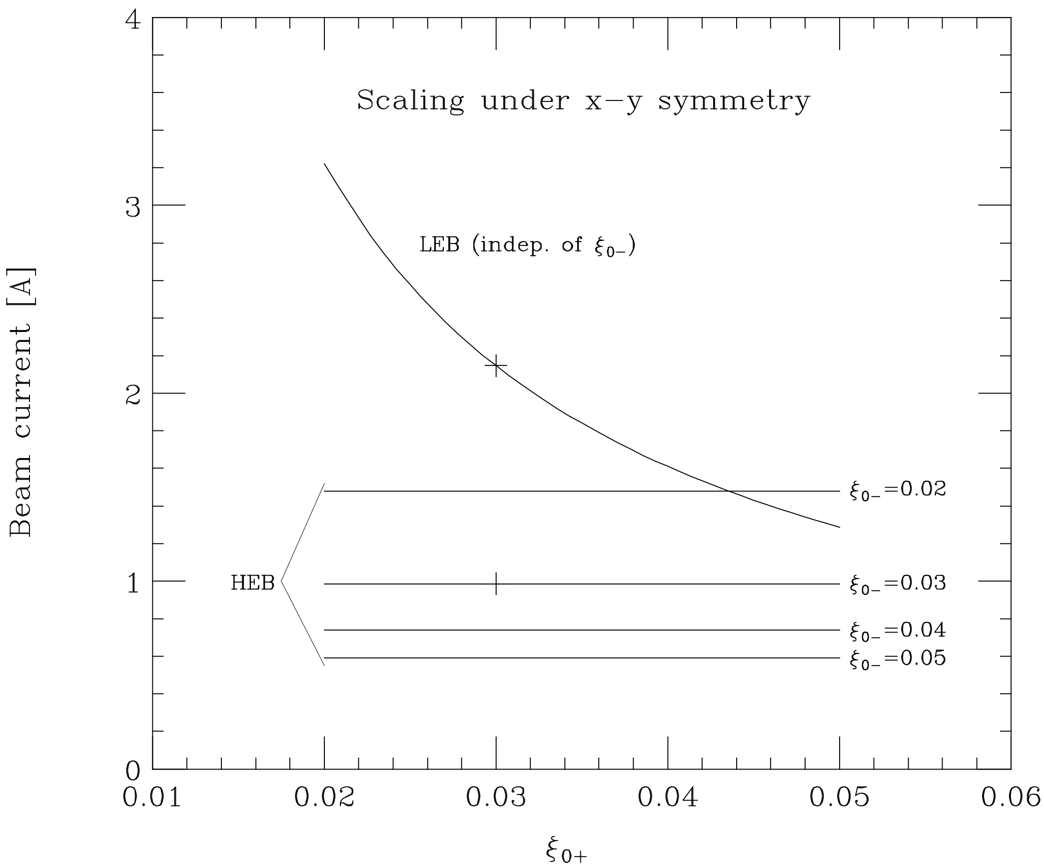


***Fig. 6. Scaling of the beam currents in the case of x-y symmetry. The cross represents the reference (CDR) values, listed in Table 1.***

## 6. Two special cases.

In Ref. [4] we used a restricted form of the scaling rules, which we now derive for the sake of completeness, in which the beam-beam parameters are further constrained so that there is only one independent variable.

### *6.1 Case 1:* $\xi_{0x} = \xi_{0y}$, $\xi_{0+} \neq \xi_{0-}$ *such that* $\xi_{0+} \cdot \xi_{0-} = \bar{\xi}_0^2$.

This is what we called "Approach A" in Ref. [4]. In this case the two nominal beam-beam parameters are equal, $\bar{\xi}_{0+} = \bar{\xi}_{0-} \equiv \bar{\xi}_0$, and the scaling rules (27) reduce to

$$
\begin{aligned}
I_\pm &= \bar{I}_\pm \times \left( \frac{\bar{\xi}_0}{\xi_{0+}} \right)^{\pm 1} \\
\sigma_{0x} &= \bar{\sigma}_{0x} = \text{independent of } \xi_{0\pm} \\
\sigma_{0y} &= \bar{\sigma}_{0y} = \text{independent of } \xi_{0\pm}
\end{aligned}
\qquad (28)
$$

*6.2 Case 2:* $\xi_{0+} = \xi_{0-}$, $\xi_{0x} \neq \xi_{0y}$ *such that* $\xi_{0x} \cdot \xi_{0y} = \bar{\xi}_0^2$.

This is what we called "Approach B" in Ref. [4]. In this case the nominal beam-beam parameters are also equal, $\bar{\xi}_{0x} = \bar{\xi}_{0y} \equiv \bar{\xi}_0$, and the scaling rules (13) and (16) reduce to

$$\begin{aligned} I_\pm &= \bar{I}_\pm \times f(\xi_{0y}) \\ \sigma_{0x} &= \bar{\sigma}_{0x} \times \frac{\xi_{0y}}{\bar{\xi}_0} f(\xi_{0y}) \\ \sigma_{0y} &= \bar{\sigma}_{0y} \times \frac{\bar{\xi}_0}{\xi_{0y}} f(\xi_{0y}) \end{aligned} \tag{29}$$

where $f(\xi_{0y})$ is the scaling function

$$f(\xi_{0y}) = \frac{1 + r_\beta}{\dfrac{\xi_{0y}}{\bar{\xi}_0} + r_\beta \dfrac{\bar{\xi}_0}{\xi_{0y}}} \tag{30}$$

## 7. Discussion.

Simulations in Ref. [4] for two earlier designs of PEP-II suggest that the beams prefer unequal beam-beam parameters. Although no serious optimization was attempted, the preference shown by the dynamics is as follows: under transparency symmetry conditions, the preferred nominal beam-beam parameters are $\xi_{0x}$ = 0.038 and $\xi_{0y}$ = 0.024; under *x-y* symmetry conditions, the preferred values are $\xi_{0+}$ = 0.026 and $\xi_{0-}$ = 0.035. Assuming that the current PEP-II design will show the same preference, then the scaling rules above allow us to determine immediately the beam currents and spot sizes for these modified beam parameters. The resultant values are found in Tables 2 and 3 below (the beta functions at the IP are the same as in Table 1).

***Table 2. Modified nominal beam-beam parameters, rms beam sizes at the IP and total beam currents assuming transparency-symmetry conditions for the beam-beam parameters.***

| | LEB ($e^+$) | HEB ($e^-$) |
|---|---|---|
| $\xi_{0x}$ | 0.038 | 0.038 |
| $\xi_{0y}$ | 0.024 | 0.024 |
| $\sigma_{0x}$ [μm] | 147.3 | 147.3 |
| $\sigma_{0y}$ [μm] | 9.327 | 9.327 |
| $r \equiv \sigma_{0y}/\sigma_{0x}$ | 0.043 | |
| $I$ [A] | 2.625 | 1.205 |
| $\mathscr{L}_0$ [cm$^{-2}$ s$^{-1}$] | $3 \times 10^{33}$ | |

***Table 3. Modified nominal beam-beam parameters, rms beam sizes at the IP and total beam currents assuming x-y-symmetry conditions for the beam-beam parameters.***

| | LEB ($e^+$) | HEB ($e^-$) |
|---|---|---|
| $\xi_{0x}$ | 0.026 | 0.035 |
| $\xi_{0y}$ | 0.026 | 0.035 |
| $\sigma_{0x}$ [μm] | 150.7 | 150.7 |
| $\sigma_{0y}$ [μm] | 6.030 | 6.030 |
| $r \equiv \sigma_{0y}/\sigma_{0x}$ | 0.04 | |
| $I$ [A] | 2.477 | 0.845 |
| $\mathscr{L}_0$ [$cm^{-2}$ $s^{-1}$] | $3 \times 10^{33}$ | |

A comparison of Table 2 with Table 1 shows that, if one would want to modify the beam parameters under transparency symmetry, one must pay the price of an increased beam current in both beams. However, there is a slight decrease in the horizontal spot size and a slight increase in the vertical spot size, both of which are favorable.

A comparison of Table 3 with Table 1 shows that, if one would want to modify the beam parameters under *x*-*y* symmetry, one must pay the price of an increased LEB current. In this case, however, there is a favorable decrease in the HEB current, while the spot sizes remain practically unchanged from the reference values.

## 8. Conclusions.

We have presented the scaling rules for the spot sizes and beam currents applicable when the beam-beam parameters vary away from their reference values. Only a dynamical calculation of the beam-beam effect will allow a determination for the preferred values of the nominal beam-beam parameters; our scaling rules allow a straightforward determination of the modified nominal emittances and beam currents, should further research indicate the need for such a modification.

## 9. References.


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4. J. R. Eden and M. A. Furman, "Simulation of the Beam-Beam Interaction for PEP-II with Unequal Beam-Beam Parameters," PEP-II/AP Note 6-92, ESG Tech Note-216, February, 1993.

5. M. A. Furman, "Luminosity Formulas for Asymmetric Colliders with Beam Symmetries," ABC note 25/ESG tech note 163, February, 1991 (rev. Sept., 1991).