Chapter 4. OPERATIONAL CONSIDERATIONS

4.1 LUMINOSITY

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Definition When two bunches ($+$ and $-$) having $N_\pm$ particles and distributions $\rho_\pm(\mathbf{x},t)$ (normalized by $\int d^3\mathbf{x}\rho_\pm(\mathbf{x},t) = N_\pm$) collide, the single-collision luminosity $\mathcal{L}_{sc}$ is defined as the number of reaction events produced per unit reaction cross section, and is given by the overlap integral [1, 2]

$$\mathcal{L}_{sc} = \frac{1}{c}\int d^3\mathbf{x}dt\,\rho_+(\mathbf{x},t)\rho_-(\mathbf{x},t) \times\sqrt{c^2(\mathbf{v}_+ - \mathbf{v}_-)^2 - (\mathbf{v}_+\times\mathbf{v}_-)^2} \quad (1)$$

where $\mathbf{v}_+(\mathbf{v}_-)$ is the common velocity of all the particles in bunch $+(-)$. Eq.(1) is a relativistic invariant, has dimensions of $1/\mathrm{area}$, and is valid for arbitrary velocities $\mathbf{v}_\pm$. (Generalization to the case when the velocity distributions are not homogeneous is given in Ref.[3].)

For a storage ring collider with bunch spacing s_B, bunches collide periodically with frequency $f_c = \beta c/s_B$. For a linear collider, $f_c =$ (repetition rate)$\times$(number of bunches per bunch train). The peak luminosity is given by $\mathcal{L} = \dot{N}/\sigma = f_c\mathcal{L}_{sc}$ [4]. It is traditionally expressed in cgs units, cm^{-2}s^{-1}.

Table 1 gives expressions for $\mathcal{L}$ in various situations for head-on collisions and σ_z small compared to β_x^*, β_y^*. These expressions are valid even with nonzero dispersion at the IP, unless otherwise noted. For initial estimates using Tab.1, we use the nominal emittances and beam sizes, but these nominal values generally change with the beam-beam force and the luminosity should be modified accordingly [5].

In the y plane, the beam-beam tune shift parameter of an on-axis particle in the positron beam due to its interaction with the opposing beam is

$$\xi_{y,+} = \frac{r_e N_- \beta^*_{y,+}}{2\pi\gamma_+\sigma^*_{y,-}(\sigma^*_{x,-}+\sigma^*_{y,-})} \quad (2)$$

Expressions for the remaining tune shift parameters are obtained by $x \leftrightarrow y$ and/or $+\leftrightarrow -$.

Transparency symmetry In a two-ring e$^+$e$^-$ collider, beam parameters need not be identical in both rings. To restrict the available parameter space, it has been suggested [6, 7, 8] that parameters be chosen to mimic the situation in a symmetric collider. The "transparency" conditions commonly adopted by designers of two-ring colliders include: (i) pairwise equality of beam-beam tune shift parameters ($\xi_{x,+} = \xi_{x,-}$; $\xi_{y,+} = \xi_{y,-}$); (ii) pairwise equality of beam sizes ($\sigma^*_{x,+} = \sigma^*_{x,-}$; $\sigma^*_{y,+} = \sigma^*_{y,-}$); (iii) equality of tune modulation amplitudes associated with synchrotron oscillations ($(\sigma_z\nu_s/\beta^*_{x,y})_+ = (\sigma_z\nu_s/\beta^*_{x,y})_-$; and sometimes (iv) equality of radiation damping decrements for the two rings.

Optimal coupling Choosing parameters such that all four beam-beam parameters are equal is called "optimal coupling." This case requires [5, 6]

$$\left(\frac{\sigma_y^*}{\sigma_x^*}\right)_+ = \left(\frac{\sigma_y^*}{\sigma_x^*}\right)_- = \left(\frac{\beta_y^*}{\beta_x^*}\right)_+ = \left(\frac{\beta_y^*}{\beta_x^*}\right)_- = \left(\frac{\epsilon_y}{\epsilon_x}\right)_+ = \left(\frac{\epsilon_y}{\epsilon_x}\right)_- \equiv r \quad (3)$$

Alternative expressions Because the luminosity in a circular collider is limited by the value of the ξ, it is useful to write $\mathcal{L}$ explicitl in terms of ξ as seen in the third row of Tab.1. Here E and I are the beam energy and total beam current in one ring and $K = 1/(2e^3) = 1/(2er_em_ec^2)$. With E in GeV, I in A, β_y^* in cm, and $\mathcal{L}$ in cm^{-2}s^{-1}, we have $K = 2.17\times 10^{34}$. The symbol $(\ )_{+,-}$ means that the enclosed parameters may be taken from either beam, on account of the transparency conditions.

For a linear collider,

$$\mathcal{L} = \frac{H}{4\pi E}\frac{N}{\sigma_x^*}\frac{P}{\sigma_y^*} \quad (4)$$

Table 1: Head-on luminosity expressions for short upright gaussian bunches.

Expression for $\mathcal{L}$	Conditions for validity
$\frac{N_+N_-f_c}{2\pi\sqrt{(\sigma^{*2}_{x,+}+\sigma^{*2}_{x,-})(\sigma^{*2}_{y,+}+\sigma^{*2}_{y,-})}}$	general
$\frac{N_+N_-f_c}{4\pi\sigma^*_x\sigma^*_y}$	$\sigma^*_{x,+}=\sigma^*_{x,-}\equiv\sigma^*_x,\ \sigma^*_{y,+}=\sigma^*_{y,-}\equiv\sigma^*_y$
$K(1+r)\xi_y\left(\frac{EI}{\beta^*_y}\right)_{+,-}$	$\sigma^*_{x,+}=\sigma^*_{x,-}\equiv\sigma^*_x,\ \sigma^*_{y,+}=\sigma^*_{y,-}\equiv\sigma^*_y,$ $\xi_{x,+}=\xi_{x,-}\equiv\xi_x,\ \xi_{y,+}=\xi_{y,-}\equiv\xi_y$
$K(1+r)\left(\xi\frac{EI}{\beta^*_y}\right)_{+,-}$	$\sigma^*_{x,+}=\sigma^*_{x,-}\equiv\sigma^*_x,\ \sigma^*_{y,+}=\sigma^*_{y,-}\equiv\sigma^*_y,$ $\xi_{x,+}=\xi_{y,+}\equiv\xi_+,\ \xi_{x,-}=\xi_{y,-}\equiv\xi_-$
$\frac{Nf_c\gamma\xi}{r_0\beta^*}$	$\sigma^*_{x,+}=\sigma^*_{x,-}=\sigma^*_{y,+}=\sigma^*_{y,-},$ $\beta^*_{x,+}=\beta^*_{x,-}=\beta^*_{y,+}=\beta^*_{y,-}\equiv\beta^*,$ $N_+=N_-\equiv N,\ E_+=E_-\equiv E$
$\frac{N^2f_c}{4\pi\epsilon\beta^*}$	$\epsilon_{x,+}=\epsilon_{x,-}=\epsilon_{y,+}=\epsilon_{y,-}\equiv\epsilon,$ $\beta^*_{x,+}=\beta^*_{x,-}=\beta^*_{y,+}=\beta^*_{y,-}\equiv\beta^*,$ $N_+=N_-\equiv N,\ E_+=E_-\equiv E,$ $D^*_{x,\pm}=D^*_{y,\pm}=0$
$\frac{\pi f_c\gamma^2\epsilon_x\xi_x\xi_y(1+r)^2}{r_0^2\beta^*_y}$	$\epsilon_{x,+}=\epsilon_{x,-}\equiv\epsilon_x,\ \sigma^*_{y,+}=\sigma^*_{y,-},$ $\beta^*_{x,+}=\beta^*_{x,-}\equiv\beta^*_x,\ \beta^*_{y,+}=\beta^*_{y,-}\equiv\beta^*_y,$ $N_+=N_-\equiv N,\ E_+=E_-\equiv E,$ $D^*_{x,\pm}=0$

where H is the pinch enhancement factor, $N=N_+=N_-$ and P is the average beam power. The factor N/σ^*_x determines the number of beamsstrahlung photons emitted (constrained by background considerations); the factor P/σ^*_y represents the major technical challenge—providing high beam power and very small bunch size.

Reductions to luminosity When $\sigma_z \gtrsim \beta^*$, the loss in luminosity due to geometrical (hourglass) effect for Gaussian beams is [9]

$$R(t_x,t_y)\equiv\frac{\mathcal{L}}{\mathcal{L}_0} = \int_{-\infty}^{\infty}\frac{dt}{\sqrt{\pi}}\frac{\exp(-t^2)}{\sqrt{(1+t^2/t_x^2)(1+t^2/t_y^2)}} \quad (5)$$

with

$$t_x^2=\frac{2(\sigma^{*2}_{x,+}+\sigma^{*2}_{x,-})}{(\sigma^2_{z,+}+\sigma^2_{z,-})\left(\sigma^{*2}_{x,+}/\beta^{*2}_{x,+}+\sigma^{*2}_{x,-}/\beta^{*2}_{x,-}\right)}$$

and correspondingly for t_y. The nominal luminosity, $\mathcal{L}_0$, is that represented by Tab.1. See Fig.1 [9].

Another reduction factor comes from a non-zero horizontal crossing angle. For the symmetric-collider case with $\sigma^*_y \ll \sigma^*_x$ we obtain [10]

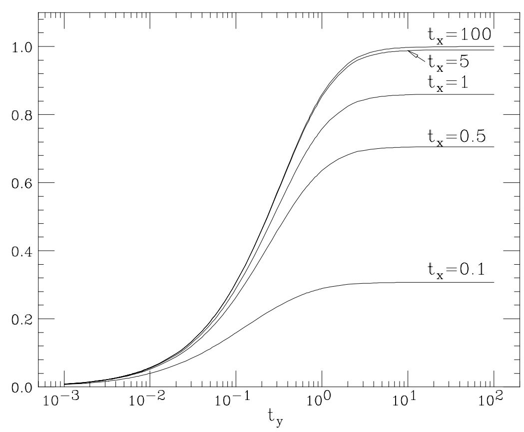

Figure 1: Hourglass reduction factors, Eq.(5).

$$R_L\equiv\frac{\mathcal{L}}{\mathcal{L}_0}=\sqrt{\frac{2}{\pi}}ae^bK_0(b) \quad (6)$$

$$a=\frac{\beta^*_y}{\sqrt{2}\sigma_z},\ b=a^2\left[1+\left(\frac{\sigma_z}{\sigma^*_x}\tan\phi\right)^2\right] \quad (7)$$

where K_0 is a Bessel function and ϕ is half the crossing angle. When $\sigma_z \ll \beta^*_y$, Eq.(6) reduces to

[3]

$$R_L = \left[1 + \left(\frac{\sigma_z}{\sigma_x^*}\tan\phi\right)^2\right]^{-1/2} \qquad (8)$$

If the beams are, in addition, offset transversely by δx and δy, Eq.(8) acquires an extra factor of

$$\exp\left[-\frac{(\delta x/2)^2}{\sigma_x^{*2}\cos^2\phi + \sigma_z^2\sin^2\phi} - \left(\frac{\delta y}{2\sigma_y^*}\right)^2\right] \qquad (9)$$

Optimization of the average luminosity Following injection, the luminosity decays in time due to particle losses from various sources. If it takes a time t_f to refill the beams, during which time the beams are not colliding, one often wants to determine the length of the luminosity run t_c that leads to the largest average luminosity. If we make the approximation $\mathcal{L}(t) = \mathcal{L}_0 \exp(-t/\tau)$ where τ is the characteristic lifetime, then the average luminosity is given by (The exponential decay is a convenient approximation; for more details, see Sec.3.4.1.)

$$\langle\mathcal{L}\rangle = \frac{\int_0^{t_c} dt\mathcal{L}(t)}{t_c + t_f} = \mathcal{L}_0\tau\frac{1 - e^{-t_c/\tau}}{t_c + t_f} \qquad (10)$$

If t_f is independent of the number of particles left in the machine at the end of the luminosity run, the equation for t_c that maximizes $\langle\mathcal{L}\rangle$ that follows from (10) is [11]

$$e^x = 1 + x + a \qquad (11)$$

where $x = t_c/\tau$ and $a = t_f/\tau$. Given a, this equation can be readily solved numerically by iteration. An approximate expression for the solution is

$$x \simeq \log\left(1 + \sqrt{2a} + a\right) \qquad (12)$$

whose relative error is at most $\sim$ 7%, and this worst case occurs for $a \simeq 1.1$. Thus, if the condition (11) is satisfied, the maximum average luminosity is

$$\langle\mathcal{L}\rangle_{\max} = \mathcal{L}_0 e^{-x} \simeq \frac{\mathcal{L}_0}{1 + \sqrt{2a} + a} \qquad (13)$$

If the filling time does depend on the number of particles left in the machine at the end of the luminosity run, the optimal condition is, of course, more complicated [11], although a similar analysis is applicable.

Integrated Luminosity In most experiments, it is the integrated luminosity that serves as the figure of merit for a collider. To account for down time, injection, beam lifetimes, etc., one experimental "year" is taken by convention to be 10^7 s. Then, the expected integrated luminosity for a collider delivering a peak luminosity of $\mathcal{L} = 1 \times 10^{33}$ cm^{-2}s^{-1} would be $\mathcal{L}_i = 1 \times 10^{40}$ cm^{-2} or 10 fb^{-1} (1 b$\equiv 10^{-24}$ cm^2). Figs.2, 3 and 4 show the luminosity history of CESR, SLC and TEVATRON (note that the scales are different in the three cases; also, CESR and the the SLC use different definitions of peak luminosity).

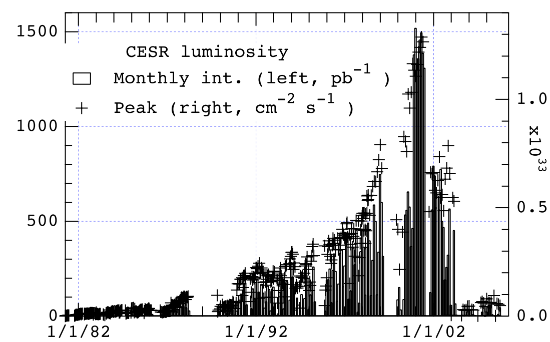

Figure 2: Luminosity history of CESR. Beam energy varied over time. 2000 and early 2001: $E = 5.3$ GeV (Υ(4S) operation). Later: $E = 4.7 - 5.17$ GeV. Starting in early 2003: CESR-c operation at $E = 1.9$ GeV. Data courtesy D. Rice

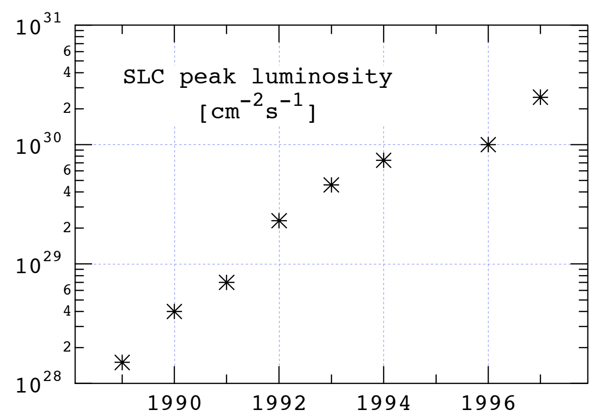

Figure 3: Peak luminosity of the SLC. Data for 1989 and 1990 were taken with the Mark II detector, subsequent data with the SLD. Data from a short run in 1995 is combined with a long run in 1994.

Monochromatization scheme Recent designs for τ-charm factories call for a "monochromati-

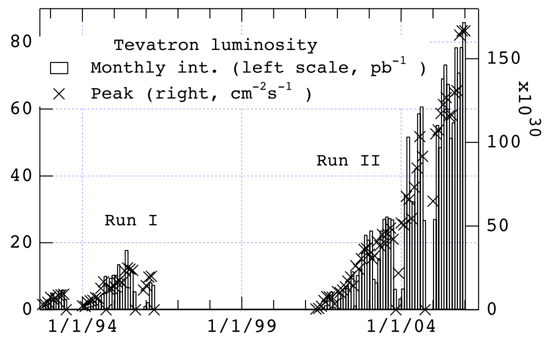

Figure 4: Luminosity history of the Tevatron. Data courtesy J. Crawford and V. Shiltsev.

zation option" with large y dispersion such that $D^*_{y,+} = -D^*_{y,-}$. This choice induces a correlation between the y position of the particles and their energy in such a way that e^+'s with higher-than-average energy preferentially collide with e^-'s with lower-than-average energy and vice versa, so that the energy spread of the c.m. of any given colliding e^+e^- pair is effectively reduced relative to the standard (zero dispersion) case, hence the name "monochromatization" [12]. The purpose of this scheme is to enhance the production of narrow resonances such as the J/ψ, whose width-to-mass ratio, $\Gamma/mc^2 = 2.8 \times 10^{-5}$, is small compared to the energy spread of the beam, typically $\sigma_\delta \lesssim 10^{-3}$. The improved energy resolution also allows detailed measurement of thresholds and branching rates in the e^+e^- c.m. energy range $w =$ 3-5 GeV [14, 15].

Since the production cross-section $\sigma(w)$ for the process $e^+ + e^- \to J/\psi$ near resonance has a significant variation as a function of w, the energy distributions of the particles in the beams are important, and the event rate is not given by $\mathcal{L}\sigma$ but rather by

$$\dot{N} = \int_0^\infty dw\, \Lambda(w)\sigma(w) \tag{14}$$

where $\Lambda(w)$ is the "differential luminosity" [14].

$\Lambda(w)$ is given by Eq.(1), except that the distributions $\rho_\pm$ must be augmented to include the dependence on E_+ and E_- of the colliding e^+e^- pair. The overlap integral is carried out subject to the constraint of fixed $w \simeq (4E_+E_-)^{1/2}$. If the two beams have the same central energy E_0 and the dispersions at the IP satisfy $D^*_{y,+} = -D^*_{y,-} \equiv D^*_y$, $D^*_{x,+} = -D^*_{x,-} = 0$, then for short gaussian bunches [14],

$$\Lambda(w) = \frac{\mathcal{L}_0}{\sqrt{2\pi}\sigma_w} e^{-\lambda^2(w-2E_0)^2/2\sigma_w^2} \tag{15}$$

where $\sigma_w = \sqrt{2}\sigma_\delta E_0$ and λ is the "monochromatization factor"

$$\lambda = \sqrt{1 + \frac{(D^*_y\sigma_\delta)^2}{\beta^*_y\epsilon_y}} \tag{16}$$

In Eq.(15) $\mathcal{L}_0$ is the luminosity in the absence of dispersion. The factor λ is chosen to be large, $\lambda \sim 10$. Therefore, the c.m. energy resolution is $\sigma_w/\lambda \ll \sigma_w$.

The luminosity is $\mathcal{L} = \int_0^\infty dw\, \Lambda(w) = \mathcal{L}_0/\lambda$, which is $\ll \mathcal{L}_0$. In fact, the resonant production rate is not reduced; only the nonresonant background is reduced by the λ-factor, so that the monochromatization scheme enhances the relative resonance production over background by a factor of λ. To see this, let $\sigma(w) = B + A\delta(w - mc^2)$ with $A \propto \Gamma$ the area under the resonance and B the nonresonant background; we get

$$\dot{N} = \frac{\mathcal{L}_0 B}{\lambda} + \frac{\mathcal{L}_0 A}{\sqrt{2\pi}\sigma_w} \tag{17}$$

The factor λ decrease in the nonresonant luminosity can be recovered by increasing ξ_y or ξ_x or N (see, e.g., any entry except the 6th in Tab.1).

If the monochromatization conditions are subject to small errors and $\lambda \gg 1$, the perturbed factor λ' is given by [16]

$$\frac{1}{\lambda'^2} = \frac{1}{\lambda^2}\left[1 + \frac{\Delta\sigma^*_{y0}}{\sigma^*_{y0}} + \frac{\Delta D^*_y}{D^*_y} + \frac{\lambda^2}{4}\left(\frac{\Delta D^*_y}{D^*_y}\right)^2\right] \tag{18}$$

where $\sigma^*_{y0} = (\beta^*_y\epsilon_y)^{1/2}$.

In the initial proposal of the monochromatization scheme [12], the vertical dispersion was supposed to be achieved with electrostatic separators, which naturally pull the beams in opposite directions. More recent multibunch "factory" designs call for separate rings for the two beams, and therefore other options become available such as electrostatic skew-quadrupole magnets [13] owing to the decoupled optics of the two rings. The lattice design must be flexible in order to accommodate the standard as well as the monochromatization configurations. The usual "factory-like" constraints arising from multibunch operation must be met, such as adequate radiation protection in the interaction region, prompt beam separation, acceptable level of background in the detectors, etc. The optics must provide for low emittance and the beam-beam parameter is chosen in the traditional range $0.03 - 0.05$. In addition to these standard requirements, of course, the dispersion at the IP must be nonzero, and is typically chosen in the range $D_y^* = 0.3 - 0.5$ m [14, 15, 17]. The combination of these constraints and the beam-beam effect [18] strongly suggest that the beta-functions at the IP must satisfy $\beta_x^* \ll \beta_y^*$, which is opposite from the standard case.

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4.2 BRIGHTNESS

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Particle density in phase space is generally referred to as the brightness distribution. The brightness distribution plays an important role in beam transport calculation and an invariant characterization of the source strength. The concept applies to both particle and photon beams.

4.2.1 Particle Beam

Brightness Distribution The brightness distribution $\mathcal{B}$ is the density in 6-D phase space [1] (x, p_x, y, p_y, t, E) where t is the arrival time, E is the kinetic energy canonical conjugate to t. When the system has a Hamiltonian, $\mathcal{B}$ is invariant along each particle trajectory in an accelerator.

When $\gamma \gg 1$, the phase space coordinates $\approx (x, \gamma x', y, \gamma y', z, \Delta\gamma)$, where z is the particle position relative to beam center, and $\Delta\gamma = \gamma - \gamma_0$ with γ_0 energy of the reference particle. Without acceleration, another convenient set of the phase space coordinates is $(x, x', y, y', z, \Delta\gamma)$. $\mathcal{B}$ is defined as the density in the appropriate phase space.

Assume $\gamma \gg 1$ with no acceleration. Assume $\mathcal{B}$ factorizes in the three dimensions, and consider x-dimension, (Extension to general case is straightforward.)

$$\mathcal{B}(x, x'; s) = \frac{d^2F}{dxdx'} \tag{1}$$

where F may be considered as the flux or longitudinal particle density. The $\mathcal{B}$ distributions at two different s are related by the coordinate transformation between them,

$$\mathcal{B}\left(x_2, x_2'; s_2\right) = \mathcal{B}\left(x_1, x_1'; s_1\right) \tag{2}$$

$$\begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = \mathbf{M}^{-1} \begin{pmatrix} x_2 \\ x_2' \end{pmatrix}$$

The spatial and the angular densities of the flux are

$$S(x; s) = \frac{dF}{dx} = \int B(x, x'; s)dx' \tag{3}$$

$$A(x'; s) = \frac{dF}{dx'} = \int B(x, x'; s)dx \tag{4}$$

$$F = \int S(x; s)dx = \int A(x'; s)dx' = \int B(x, x')dxdx'$$

S and A are not invariant along the particle trajectory. In the absence of aperture, F is conserved and is an invariant characterization of the global strength of the beam.

Brightness For a well-designed beam, $\mathcal{B}$ is a smooth function peaked at the phase space origin. Thus $\mathcal{B}$(origin) is often referred to as the brightness. A related quantity is the emittance (phase space area). The brightness is flux divided by emittance. There are different definitions of the emittance, and hence in the brightness. One definition is the rms emittance [2] $\epsilon_x = \sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$ where $\langle\rangle$ means averaging with $\mathcal{B}$ as the weight function.