SSC-N-481

BUNCH TILT DUE TO TRANSVERSE IMPEDANCES IN THE SSC

M. A. FURMAN
SSC Central Design Group
March 1988

ABSTRACT

We evaluate the bunch tilt produced by localized transverse impedances for the SSC. Even a conservative estimate yields a tiny result, so that this effect has negligible detrimental effect on the luminosity.

1. Introduction

Among the many possible effects that impedances have on a beam, we consider here only one such effect, which is stated as follows: Consider a bunch traversing a point in the ring where there exists a localized transverse impedance; if the bunch has a nonzero closed orbit offset its wake field produces a transverse force. The kick is not uniform along the length of the bunch, so that, typically, the head and tail experience different closed orbit distortions due to this kick. In other words, the bunch tilts. The oncoming bunch also tilts, and since they are very needle-like, there is concern that the luminosity is potentially degraded if the tilt is large enough.[1]

We describe below a reasonable statistical estimate of the tilt at an IP giving an *rms* value for the head-tail closed orbit difference of about 1.5 milli-microns, while a pessimistic estimate yields about 30 milli-microns. In either case this is much smaller than the bunch transverse size of 5 microns, so the effect is negligible.

2. Formulas and Assumptions

We consider a bunch moving at the speed of light c in a ring of radius R and pipe radius b with angular frequency ω_0. This bunch traverses a single localized impedance with a resonator form

$$Z_\perp(\omega) = \sum_{\text{modes}} \frac{R_\perp\left(\frac{\omega_0}{\omega}\right)}{1 + iQ\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)} \tag{2.1}$$

where Q, ω_r and $R_\perp$ are the quality factor, resonance frequency and transverse resistance of the mode. We assume also that $R_\perp$ is given, in terms of the shunt impedance R_S, by the resistive wall formula

$$R_\perp = \frac{2R}{b^2} R_S \tag{2.2}$$

If the bunch has uniform charge distribution with N_B particles and full angular size 2α then, in leading approximation in the bunch current, the tail suffers a kick given by

$$\Delta x' = N_B \left(\frac{m_0 c^2}{E}\right)\left(\frac{r_0}{\sigma_z}\right) \sum_{\text{modes}} \left(\frac{c\, R_\perp\, x_{\text{co}}}{Q\, h}\right) T(h\alpha, Q) \tag{2.3}$$

while the head of the bunch is not kicked.[1] Here m_0 and r_0 are the proton mass and classical radius, E is the beam energy, σ_z the *rms* bunch length, x_{co} the closed orbit displacement at the location of the impedance, and $h \equiv \omega_r/\omega_0$ (note that $\alpha = \sqrt{3}\sigma_z/R$ for a uniformly charged bunch). The "tilt function" T is given by

$$T(x,Q) = 1 - \mathrm{e}^{-x/Q}\left[\frac{\sin(2xF)}{2QF} + \cos(2xF)\right] \tag{2.4}$$

where $F \equiv \sqrt{1 - 1/4Q^2}$. Note that the effect is significant only for broad-banded impedances.

If there are many locations in the ring with localized impedances, the net effect is obtained by the addition of the individual contributions. Since we are only interested in an *rms* estimate, we add the contributions with appropriate statistical weights, as described in more detail below. Thus the effect of the kicks translate into a head-tail closed orbit difference at an IP whose *rms* estimate is

$$\Delta x = \sum_i w_i \sqrt{\frac{\beta^* \beta_i}{2}}\, \Delta x' \tag{2.5}$$

where β^* and β_i are values of the beta function at the IP and at the location of the impedance, respectively (the factor $1/\sqrt{2}$ is the *rms* value of the phase advance factor), and w_i is the statistical weight for each contribution.

3. Results and Discussion

For a numerical application to the SSC we consider resonant impedances arising from four sources, which are dominant for the present purposes: beam position monitors, RF cavities, sliding contact bellows and inner bellows, with resonator parameters given in the tables in the Appendix.[2]

For the BPMs, sliding contact bellows and inner bellows we assume a beta function value $\beta_i = 225$ m, while for the cavities we assume $\beta_i = 625$ m. Since the 40 cavities are clustered together in a region that is short compared with the betatron wavelength, we take their weight to be $w_i = 40$. The 850 BPMs, 5000 sliding contact bellows and 4.54×10^5 corrugations of inner bellows are more or less evenly distributed around the ring, so we assume the weights to be $w_i = \sqrt{850}$, $w_i = \sqrt{5000}$ and $w_i = \sqrt{4.54 \times 10^5}$, respectively. In all cases we take the closed orbit offset to be $x_{\mathrm{co}} = 1$ mm.

For an IP with $\beta^* = 0.5$ m we obtain a head-tail displacement $\Delta x = 1.4 \times 10^{-9}$ m, which should be compared with the bunch width $\sigma^* = 5 \times 10^{-6}$ m. The tilt is therefore negligible, and so it is its effect on the luminosity. The result is dominated by the BPMs' impedance; even if all the BPMs were added together coherently (*i.e.*, with a weight factor $w_i = 850$ instead of $\sqrt{850}$), the resulting displacement would be $\Delta x \simeq 30 \times 10^{-9}$ m, which is still well below a worrisome value.

The bunch tilt effect was originally studied for PEP,[1] where it was found to be small, $\Delta x \simeq 20 \times 10^{-6}$ m. A simple minded analysis indicates that the effect discussed here, described more precisely by the ratio $\Delta x/\sigma^*$, is scale invariant with energy. The fact that it is more than a factor of 1000 smaller for the SSC than for PEP is a consequence of the much smaller bunch current (more than a factor of 100 smaller) and also the smaller impedance per unit length of the ring.

ACKNOWLEDGEMENTS

I am grateful to Alex Chao for discussions.

APPENDIX

Below are the parameters for the four sources of impedance considered.[2] Those of the cavities represent an effective broadband superposition of their modes[1] for one single cavity (there is a total of 40 cavities). Those of the sliding contact bellows refer to one single bellows, of which there are 850, and those of the inner bellows are for one single corrugation, of which there is a grand total of 4.54×10^5.

Table 1. Parameters of a single BPM

f_R [GHz]	R_S [Ω]	Q
2.9	141	1

Table 2. Effective parameters for one cavity

f_R [GHz]	R_S [Ω]	Q
1.5	8.98	1.5
6	3.42	0.5

Table 3. A single sliding contact bellows

f_R [GHz]	R_S [Ω]	Q
6.2	20	6
16	30	1

Table 4. A single corrugation of an inner bellows

f_R [GHz]	R_S [Ω]	Q
12.3	13.4	4
49.0	3.52	20

REFERENCES

1. A. W. Chao and S. Kheifets, "Orbit Distortion and Beam Tilt Caused by Transverse Wake Fields," PEP-Note-365, September 1981.
2. SSC Central Design Group, "Superconducting Supercollider Conceptual Design Report," SSC-SR-2020, March 1986.