Linear Space-Charge Tune Shifts for the SSC and its Injectors

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Abstract

We evaluate the incoherent linear space-charge tune shift, including the effect of image charges, for the SSC and its boosters at their respective injection energies. The effect of the images is significant only for the SSC. The tune shifts for the LEB and MEB are comparable and substantial, while the tune shifts for the HEB and SSC are small.

1 Assumptions and Formulas

We assume that all the bunches in each ring are identical, have Gaussian charge distribution in all three dimensions, and we compute the tune shift of the particle at the center. If we neglect for the moment all effects from the environment, the formula for the horizontal tune shift is [1]

$$\Delta\nu_x^{(0)} = -\frac{r_0\, N_B\, C\, \bar\beta}{2\pi\, \beta^2\, \gamma^3\, \bar\sigma_x\, (\bar\sigma_x + \bar\sigma_y)\, \sqrt{2\pi}\, \sigma_z} \tag{1}$$

and the formula for the vertical tune shift is similar to the above one with $\bar\sigma_x \leftrightarrow \bar\sigma_y$. Here $r_0 = 1.536 \times 10^{-18}$ m is the classical radius of the proton, N_B the number of particles per bunch, C the circumference of the ring, $\bar\beta$ the average of the beta-function (we assume $\bar\beta_x = \bar\beta_y \equiv \bar\beta$), β and γ are the usual relativistic factors, $\bar\sigma_x$ and $\bar\sigma_y$ the transverse *rms* bunch widths averaged over the ring, and σ_z is the *rms* bunch length in the lab frame.

The *rms* widths are given by

$$\bar\sigma_x = \sqrt{\bar\beta\epsilon + \overline{\eta^2}(\sigma_p/p)^2}\ , \qquad \bar\sigma_y = \sqrt{\bar\beta\epsilon} \tag{2}$$

where $\overline{\eta^2}$ is the averaged square of the dispersion function, p is the momentum, σ_p the *rms* momentum spread and $\epsilon = \epsilon_N/(\beta\gamma)$ the emittance (we assume $\epsilon_x = \epsilon_y \equiv \epsilon$).

The *rms* momentum spread is obtained from the longitudinal emittance $\epsilon_L \equiv \sigma_E\sigma_t$ by

$$\sigma_p = \epsilon_L/\sigma_z \tag{3}$$

and the *rms* bunch length σ_z is given, at injection, by

$$\frac{\sigma_z}{C} = \sqrt{\frac{1}{\beta}\left(\frac{\epsilon_L}{E\,T}\right)\sqrt{\frac{s}{2\pi\, h}\left(\frac{E}{eV_0}\right)}} \tag{4}$$

where $s = |\gamma_t^{-2} - \gamma^{-2}|$, γ_t is the transition gamma, h the harmonic number, V_0 the total accelerating voltage around the ring, and T, E are the revolution period and energy of the synchronous particle.

The effect of the image charges is taken into account [1] by a correction factor

$$\Delta\nu_x = \Delta\nu_x^{(0)}\, F_x \tag{5}$$

given by

$$F_x = 1 + \frac{2\,\bar\sigma_x(\bar\sigma_x + \bar\sigma_y)}{h^2}\Bigg\{\epsilon_1\left[1 + B_f\,(\gamma^2 - 1)\right] + \epsilon_2\, B_f\,(\gamma^2 - 1)\left(\frac{h}{v}\right)^2\Bigg\} \tag{6}$$

with a corresponding expression for F_y. In the above formula h is the half-height of the vacuum chamber, v the half-height of the magnet pole gap, ϵ_1 and ϵ_2 are numerical coefficients for the electric and magnetic images, respectively, and B_f is the bunching factor, defined by

$$B_f = \frac{\sqrt{2\pi}\,\sigma_z}{S_B} \tag{7}$$

where S_B is the bunch spacing.

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For the LEB and MEB we make the approximation that the width of the vacuum chamber is much larger than its height, and that the magnet poles are parallel plates. In this case the coefficients are $\epsilon_1 = \pi^2/48 = 0.206$ and $\epsilon_2 = \pi^2/24 = 0.411$. For the HEB and SSC the chamber and magnet poles are round; if the beam were exactly centered, the coefficiets ϵ_1 and ϵ_2 would vanish. However, because of the closed orbit error, this is not quite true. In this case the coefficients are given by[1]

$$\epsilon_1 = \frac{(\Delta x)^2}{2\,h^2}, \qquad \epsilon_2 = \frac{(\Delta x)^2}{2\,v^2} \tag{8}$$

where Δx is the *rms* closed orbit error, which we take to be $\Delta x = 1$ mm (this is an overestimate).

We assume parameters [2] at injection as displayed in Table 1. The results, including the tune shifts $\Delta\nu_x$ and $\Delta\nu_y$, are displayed in Table 2. It should be noted that the LEB lattice is being redesigned [3] so that the resulting tune shifts may be substantially different from the ones quoted in Table 2.

2 Discussion

The values we obtain for the tune shifts for the LEB, $\Delta\nu_x = -0.093$ and $\Delta\nu_y = -0.145$, are in good agreement with the more reliable values obtained from tracking [4], -0.089 and -0.127, respectively. This gives us confidence that the formulas we use are reasonable approximations.

It should be noted that, for the LEB, the $\overline{\eta^2}$-term is larger than the $\bar{\beta}$-term in formula (2). This makes the tune shifts, especially the horizontal one, more sensitive to the dispersion than to the emittance. The proposed new LEB lattice design [3] has much smaller dispersion so that this is no longer the case. In addition, the circumference of the new lattice is larger, and a very preliminary calculation indicates that the resulting tune shifts may be bigger than those in Table 2 by a factor of 2 or so.

The tune shifts for the LEB and MEB are substantial and comparable. This is because when going from the LEB to the MEB the decrease in value of the factor $1/\left(\beta^2\,\gamma^3\right)$ in formula (1) is not enough to compensate for the change of the other factors, all of which tend to make $\Delta\nu$ larger.

The correction factors F_x and F_y are substantial only for the SSC, and this is largely due to our conservative estimate of the closed orbit error. We are not concerned about this because the resulting tune shifts are very small anyway.

[1] We are indebted to Jackson Laslett for a discussion on this point.

Acknowledgments

We are grateful to Jackson Laslett for valuable discussions.

References

[1] L. J. Laslett, "On Intensity Limitations Imposed by Transverse Space-charge Effects in Circular Accelerators," BNL-7534 (1963) p. 324, and references therein; further discussions on the matter can be found in "Selected Works of L. J. Laslett," LBL-PUB-616, September 1987.

[2] SSC Central Design Group, "Superconducting Supercollider Conceptual Design Report," SSC-SR-2020, March 1986; "Parameter List" (CDR Attachment A), SSC-SR-2020A.

[3] L. K. Chen, "A Possible Modified Lattice for the LEB," SSC-N-412, December 1987.

[4] M. Furman, "Effect of the Space-Charge Force on Tracking at Low Energy," SSC-115, March 1987. The tracking was done with the kick code TEAPOT suitably modified to include the space-charge force, but the number of particles per bunch N_B was taken to be 7.3×10^9 instead of 1×10^{10}, as we do here. Accordingly, we scale the results for the tune shifts obtained in SSC-115 by the factor 10/7.3, which yields the quoted values -0.089 and -0.127.

Table 1: Assumed Parameters at Injection

	LEB	MEB	HEB	SSC
C [m]	249.6	1,900.8	6,000	82,944
pc [GeV]	1.22	8	100	1,000
S_B [m]	4.8	4.8	4.8	4.8
$N_B \times 10^{-9}$	10	9.2	8.7	7.3
γ_t	10.5	7.2	18.7	67.0
h	52	396	1,250	103,680
eV_0 [MeV]	0.35	0.60	1.50	20.0
ϵ_L [meV-sec]	1.6	1.6	35	35
ϵ_N [mm-mrad]	0.75	0.83	0.91	1.0
$\bar{\beta}$ [m]	13	40	50	220
$\overline{\eta^2}$ [m^2]	12	6	9	9
h [cm]	4.0	4.0	2.2	1.65
v [cm]	5.0	5.0	6.1	5.5
ϵ_1	0.206	0.206	1.0×10^{-3}	1.8×10^{-3}
ϵ_2	0.411	0.411	1.3×10^{-4}	1.7×10^{-4}

Table 2: Results

	LEB	MEB	HEB	SSC
β	0.793	0.993	1.0	1.0
γ	1.641	8.587	106.6	1,066
f_0 [kHz]	952.2	156.6	49.93	3.614
T [μsec]	1.05	6.38	20.02	276.67
σ_z [cm]	41.5	14.1	31.1	6.0
$(\sigma_p/p) \times 10^4$	9.49	4.25	3.38	1.75
$\bar{\sigma}_x$ [mm]	4.28	2.23	1.205	0.694
$\bar{\sigma}_y$ [mm]	2.74	1.97	0.653	0.454
F_x	1.01	1.03	1.02	1.39
F_y	1.01	1.03	1.01	1.25
$\Delta\nu_x$	–0.093	–0.085	–0.00031	–0.00031
$\Delta\nu_y$	–0.145	–0.096	–0.00056	–0.00043