# Bunch-Length Dependence of Power Loss for the SSC

Miguel A. Furman
SSC Central Design Group

May 1986
(minor revisions: 11/99; 2/02)

## Abstract

We evaluate the power lost by several mechanisms in the SSC as function of bunch length. The items considered are: RF cavities, inner bellows, sliding-contact bellows, beam position monitors plus kickers, resistive wall, and coherent synchrotron radiation

## 1 Introduction

We assume that the beam has $M$ bunches, each with an electric charge $Ne$, moving in an orbit of length $2\pi R$ with frequency $f_0$. As these bunches circulate around the machine they lose power due to the retarding effect of the longitudinal impedance $Z_\parallel(\omega)$. For gaussian bunches the power loss is given by [1]

$$P = M I_b^2 \sum_{p=-\infty}^{\infty} \mathrm{Re} Z_\parallel(p\omega_0)\, e^{-(p\omega_0\sigma_z/c)^2} \qquad (1)$$

where $\sigma_z$ is the rms bunch length, $\omega_0 = 2\pi f_0$ and $I_b = Nef_0$ is the bunch current.

In practice we replace the summation by an integral in the above formula. This replacement is a valid approximation provided that [2]:

1. $\omega_0\sigma_z/c \ll 1$, and
2. $Z_\parallel(\omega)$ varies little (and smoothly) over a frequency interval of size $\omega_0$.

Since the average radius is given by $R = c/\omega_0$, condition (a) is equivalent to $\sigma_z \ll R$, which is very well satisfied by the SSC parameters. Condition (b) is very well satisfied by all non-resonator impedances considered here. For a resonator impedance, condition (b) translates to $\omega_r/\omega_0 \gg Q$, which is approximately satisfied by the RF impedance parameters in Table 1 below, and very well satisfied by all other resonator impedances in this note. Thus our starting formula is

$$P = M I_b^2 \int_{-\infty}^{\infty} \frac{d\omega}{\omega_0} \mathrm{Re} Z_\parallel(\omega)\, e^{-(\omega\sigma_z/c)^2} \qquad (2)$$

which, for a simple resonator form for the impedance,

$$Z_\parallel(\omega) = \frac{R_S}{1 + iQ\left(\dfrac{\omega_r}{\omega} - \dfrac{\omega}{\omega_r}\right)} \qquad (3)$$

yields [2]

$$P = M I_b^2 R_S \times \left(\frac{C}{\sigma_z}\right) \times \frac{\mathrm{Re}(zw(z))}{\sqrt{4Q^2-1}} \qquad (4)$$

where $w(z)$ is the complex error function [3] and

$$z = \frac{\omega_r \sigma_z}{2Qc}\left(\sqrt{4Q^2-1} + i\right) \qquad (5)$$

In what follows we use certain assumed forms for the impedances for the different structures, and then use Eq. (4) to calculate the power loss. We also assume nominal SSC values for the parameters, namely a cricumference $C = 82944$ m, corresponding to an average an average orbit radius $R = C/2\pi = 13.2$ km, a revolution frequency $f_0 = 3.61$ kHz (corresponding to $\omega_0 = 2\pi f_0 = 2.27 \times 10^4$ rad/s), and $M = 17280$ bunches per ring, each having $N = 7.3 \times 10^9$ protons. These parameter values imply a bunch current $I_b = Nef_0 = 4.2\ \mu$A. All results presented for the power loss are per ring.

## 2 Impedances

### 2.1 RF Cavities

The impedance of each of the 40 RF cavities is assumed to be represented by a superposition of 19 resonator modes with parameters as given in Table 1. These parameters correspond to a PEP cavity design

Table 1: PEP cavity parameters.

| $n$ | $f_r$ [GHz] | $R_S$ [MΩ] | $Q$ |
|---|---|---|---|
| 1 | 0.3589 | 11.84 | 42900 |
| 2 | 0.5072 | 2.03 | 36500 |
| 3 | 0.8302 | 0.025 | 44400 |
| 4 | 0.8834 | 0.532 | 54500 |
| 5 | 0.9261 | 1.482 | 35800 |
| 6 | 1.1671 | 0.622 | 50600 |
| 7 | 1.2157 | 0.332 | 61900 |
| 8 | 1.4401 | 0.434 | 91500 |
| 9 | 1.4474 | 0.555 | 60700 |
| 10 | 1.5330 | 0.018 | 53500 |
| 11 | 1.5860 | 0.238 | 70800 |
| 12 | 1.6685 | 0.282 | 55300 |
| 13 | 1.7377 | 0.298 | 56300 |
| 14 | 1.8522 | 0.017 | 58300 |
| 15 | 1.9589 | 0.302 | 77800 |
| 16 | 2.065 | 0.794 | 71500 |
| 17 | 2.077 | 0.130 | 72100 |
| 18 | 2.153 | 0.046 | 83600 |
| 19 | 2.290 | 0.116 | 68900 |

obtained from URMEL [5] ($f_r = \omega_r/2\pi$ is the resonant frequency of mode $n$).

In practice, due to cavity construction errors and temperature variations, the resonant frequencies vary slightly from cell to cell, effectively broadening ("de-Qing") the resonant peaks. We take this effect into account by the making the replacements

$$\begin{aligned} R_S &\to 40R_S \quad \text{for } n = 1 \\ R_S &\to 2R_S \quad \text{for } n = 2, \cdots, 19 \\ Q &\to Q/20 \quad \text{for } n = 2, \cdots, 19 \end{aligned} \tag{6}$$

to the parameter values in Table 1, and then using Eq. 4 for each mode.

## 2.2 Inner Bellows

These are represented by a superposition of two resonators [4] with parameters given by Table 2, and the power is computed from Eq. (4). The values of the shunt impedances $R_S$ take into account the total of $4.54 \times 10^5$ corrugations for the whole ring.

Table 2: Inner bellows parameters

| $f_r$ [GHz] | $R_S$ [MΩ] | $Q$ |
|---|---|---|
| 12.3 | 6.1 | 4 |
| 49.0 | 1.6 | 20 |

## 2.3 BPMs and Kickers

The 850 BPMs and two kickers (one for injection plus one for abort) are collectively represented by one single broad-band resonator as per Table 3 [6].

Table 3: Parameters for BPMs plus kickers.

| $f_r$ [GHz] | $R_S$ [MΩ] | $Q$ |
|---|---|---|
| 2.9 | 0.12 | 1 |

## 2.4 Sliding-Contact Bellows

The impedance for the 5000 units is represented [7] by the superposition of two resonators plus a frequency-independent term. The resonator parameters are presented in Table 4 (the values of $R_S$ include the appropriate factor of 5000.)

Table 4: Sliding-contact bellows parameters

| $f_r$ [GHz] | $R_S$ [MΩ] | $Q$ |
|---|---|---|
| 6.2 | 0.10 | 6 |
| 16 | 0.15 | 1 |

The resonant contribution to the power is computed from Eq. (4). The frequency-independent part of the impedance is

$$\mathrm{Re}Z_{\|}(\omega) = \frac{Z_0}{2\pi}\log\left(\frac{b+\Delta}{b}\right) \simeq \frac{Z_0}{2\pi}\left(\frac{\Delta}{b}\right) \tag{7}$$

where $Z_0 = 4\pi/c \simeq 377\ \Omega$ is the impedance of the vacuum, $\Delta = 0.1$ mm is the gap size, and $b = 1.65$ cm is the pipe radius. From Eq. (2) we get the contribution to the power loss per unit,

$$P = MI_b^2\frac{Z_0}{\sqrt{\pi}} \times \left(\frac{\Delta}{2b}\right) \times \left(\frac{R}{\sigma_z}\right) \tag{8}$$

which is then multiplied by 5000.

### 2.5 Resistive Wall

Here the impedance is [8]

$$Z_{\|}(\omega) = (1-i)\frac{R}{b\sigma\delta(\omega)} \tag{9}$$

where $\sigma = 1.6 \times 10^{19}$ s$^{-1}$ is the cold copper conductivity, and

$$\delta(\omega) = \frac{c}{\sqrt{2\pi\sigma\omega}} \tag{10}$$

is its skin depth. From Eq. (2) we get

$$P = \Gamma(\tfrac{3}{4}) M I_b^2 Z_0 \left(\frac{\delta(\omega_0)}{2b}\right)\left(\frac{R}{\sigma_z}\right)^{3/2} \tag{11}$$

### 2.6 Coherent Synchrotron Radiation

The power radiated by $M$ bunches in a circular orbit of radius $\rho$ in the absence of any shielding is [9]

$$\begin{aligned} P_0 &= \frac{\pi\Gamma(\frac{2}{3})}{3^{1/3}\Gamma(\frac{1}{3})} M I_b^2 Z_0 \sigma_\phi^{-4/3} \\ &\simeq 1.101 M I_b^2 Z_0 \sigma_\phi^{-4/3} \end{aligned} \tag{12}$$

where $\sigma_\phi = \sigma_z/\rho$ is the angular *rms* bunch size. This expression must be modified to account for the shielding provided by the vacuum chamber and for the fact the the orbit is not a perfect circle. This last modification is approximately accounted for by a factor $\rho/R$ where $\rho = 10.1$ km is the radius of curvature of the orbit in the dipole field magnets (the factor $\rho/R \simeq 0.765$ can be thought of as the fraction of the orbit that is circular). Thus the appropriate expression is

$$P = P_0 \times \left(\frac{\rho}{R}\right) \times S \tag{13}$$

where the shielding factor $S$ is given by [10]

$$S = 3^{-1/6}\, 2^{-1/3} \frac{b}{(\rho\sigma_z^2)^{1/3}} \tag{14}$$

This last expression is presumably pessimistic because it is obtained from a simplified geometry in which the shielding is provided by two infinite parallel metallic plates. A pipe presumably provides more shielding, hence the true value for $S$ is presumably smaller than that obtained from Eq. (14).

## 3 Results and Comments

Results for the power loss per beam are presented in Fig. 1. For short bunch lengths ($\sigma_z \lesssim 5$ cm), the power loss is dominated by the contribution from the BPMs plus kickers. For longer bunch lengths ($\sigma_z \gtrsim 5$ cm) the power loss is dominated by the power lost to the fundamental mode of the RF cavities. We have *not* made allowance for the change in $N$ that must accompany a change in $\sigma_z$ in order to maintain a constant luminosity. This would not change our numbers significantly.

### Acknowledgments

I am grateful to Alex Chao for many discussions.

## References

[1] A. W. Chao, in *Physics of High Energy Particle Accelerators*, M. Month, ed., AIP Conf. Proc. **105**, p. 353 (SLAC Summer School, 1982).

[2] M. A. Furman, SSC-N-142.

[3] *Handbook of Mathematical Functions*, M. Abramowitz and I. A. Stegun, eds., Dover, 1965, p. 297.

[4] K. Bane and R. Ruth, SSC-SR-1017 (1985); J. Bisognano and K. Y. Ng, *ibid.*; R. Ruth, in *Accelerator Physics Issues for a Superconducting Super Collider*, Ann Arbor, 1983, ed. by M. Tigner, Univ. of Michigan rep. UM HE 84-1; J. M. Wang and C. Pellegrini, proc. XI Int. Conf. on High Energy Accelerators, Geneva, CERN (1980); R. Ruth and J. M. Wang, IEEE Trans. Nucl. Sci., **NS-28**, 2405 (1981).

[5] Z. D. Farkas and K. Bane, SSC-N-124.

[6] R. Shafer, SSC-SR-1017; E. Colton and T. S. Wang, *ibid.* and SSC-N-144.

[7] K. Y. Ng, SSC-SR-1017.

[8] J. D. Jackson, *Classical Electrodynamics*, 2nd. ed., J. Wiley & Sons, 1975.

[9] L. I. Schiff, Rev. Sci. Inst., **17**, 6 (1946).

[10] J. Schwinger, Phys. Rev. **75**, 1912 (1949); "On Radiation by Electrons in a Betatron", c.1945, unpublished, transcribed by M. A. Furman, LBNL-39088/CBP Note-179, July 29, 1999; J. S. Nodvick and D. S. Saxon, Phys. Rev. **96**, 180 (1954).

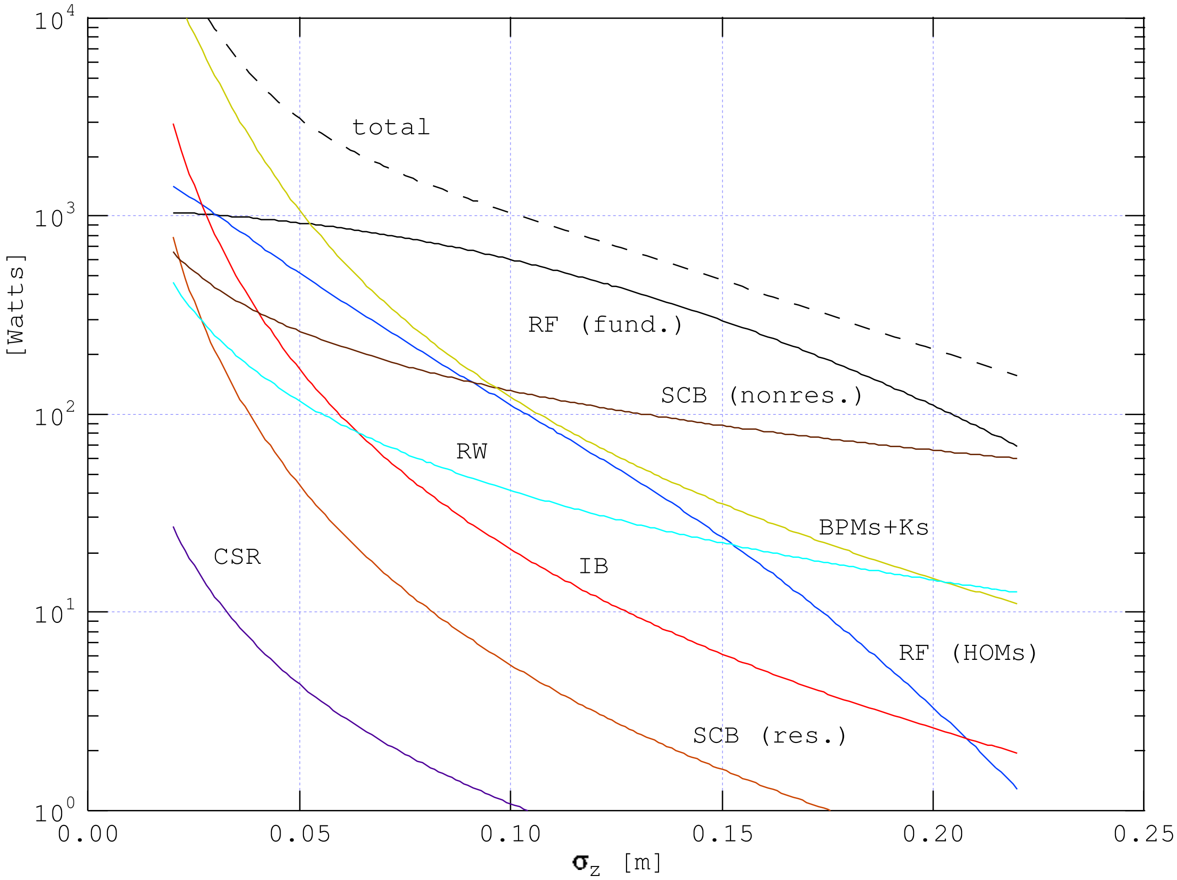


Figure 1: The various contributions to the power loss per beam. "RF (fund.)" is the power lost to the fundamental mode of the RF cavities (Eq. (4) with parameters corresponding to $n = 1$ in Table 1, modified as in Eq. (6)). "RF (HOMs)" is the aggregate power lost to the higher-order modes (Eq. (4) with parameters corresponding to $n = 2, \cdots, 19$ of Table 1 modified as in Eq. (6)). "IB" is the power lost to the inner bellows, Eq. (4) with parameters as in Table 2. "BPMs+Ks" is the power lost to the BPMs and kickers, Eq. (4) with parameters as in Table 3. "SCB" (res.) is the resonant power lost to the sliding-contact bellows (Eq. (4) with parameters as in Table 4). "SCB" (nonres.) is the power lost to the frequency-independent part of the impedance, Eq. (8)). "CSR" is the power loss due to shielded coherent synchrotron radiation, Eqs. (12–14). "total" is the sum of all contributions.