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Decoherence*†

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If a stored beam is kicked transversely by angle $\Delta x'$ (or is injected offset), its centroid betatron signal decoheres due to betatron tune spread. We define $q \equiv x/\sigma_x$ and $p \equiv (\alpha_x x + \beta_x x')/\sigma_x$ where α_x, β_x and σ_x are the lattice functions and rms beam size, respectively, at the observation point (= kick point). We assume: (1) the beam is Gaussian in (x, x') and in $\delta \equiv \Delta E/E_0$; (2) there is no x-y coupling and no synchro-betatron coupling; (3) damping, quantum excitation and the mutual interactions of the particles can be ignored; and (4) the tune dependence on amplitude and energy offset is

$$\nu = \nu_0 - \mu(q^2 + p^2) + \nu'\delta \qquad (1)$$

where $\nu' =$ chromaticity. Then the time evolution of the beam centroid is [1, 2]

$$\langle q\rangle + i\langle p\rangle = \frac{iZF(n)}{(1-i\theta)^2}\exp\left(-2\pi i n\nu_0 + \frac{Z^2}{2}\frac{i\theta}{1-i\theta}\right) \qquad (2)$$

where $n =$ turn number, $\theta = 4\pi\mu n$, $Z = \beta_x\Delta x'/\sigma_x$ and the chromatic form factor $F(n)$ is

$$F(n) = \exp\left[-2\left(\frac{\nu'\sigma_\delta}{\nu_s}\right)^2 \sin^2(\pi n\nu_s)\right] \qquad (3)$$

where $\nu_s =$ synchrotron tune. The second moments after the kick are

$$\begin{pmatrix}\langle q^2\rangle \\ \langle qp\rangle \\ \langle p^2\rangle\end{pmatrix} = \left(1 + \frac{Z^2}{2}\right)\begin{pmatrix}1\\0\\1\end{pmatrix} + \qquad (4)$$

$$\frac{Z^2F^2(n)}{2(1+4\theta^2)^{3/2}}\exp\left(-\frac{2Z^2\theta^2}{1+4\theta^2}\right)\begin{pmatrix}-\cos\psi\\ \sin\psi\\ \cos\psi\end{pmatrix},$$

$$\psi \equiv 4\pi n\nu_0 - \frac{Z^2\theta}{1+4\theta^2} - 3\tan^{-1}2\theta \qquad (5)$$

The normalized rms size is $\sigma_x(n)/\sigma_x(0) = (\langle q^2\rangle - \langle q\rangle^2)^{1/2}$. Note that $\langle q^2\rangle + \langle p^2\rangle = 2 + Z^2 =$ constant. The amplitude $A = (\langle q\rangle^2 + \langle p\rangle^2)^{1/2}$ of the beam centroid is

$$A(n) = \frac{ZF(n)}{1+\theta^2}\exp\left(-\frac{Z^2\theta^2}{2(1+\theta^2)}\right) \qquad (6)$$

Long after the kick, $\theta \gg 1$, the centroid amplitude decoheres as $A \sim \theta^{-2}$, while the rms beam size approaches an equilibrium $\sigma_x(\infty)/\sigma_x(0) = (1 + Z^2/2)^{1/2}$.

As time elapses, $F(n)$ periodically comes back to its peak value of unity. Therefore, if $\mu = 0$ (i.e., $\theta = 0$), the beam centroid "recoheres" with the synchrotron period. This effect provides a way [3] to measure the product $\nu'\sigma_\delta$ (assuming $\nu_s \ll 1$). If $\mu \neq 0$, the recoherence is still partially present.

The formulas above apply to 1-D. Extension to 2-D, including x-y coupling in the tune dependence with amplitudes, is addressed in [4]. Ref. 5 treats the decoherence phenomenon including synchrobetatron coupling, damping and quantum excitation. Ref. 6 applies the canonical Hamiltonian perturbation formalism to 2-D decoherence in the presence of an arbitrary nonlinear tune dependence on amplitudes; this formalism allows computing the decoherence rate of a beam trapped in a resonant island. Ref. 2 presents data on the dependence of decoherence rate on beam intensity in the SLC (in particular, the dependence on the sign of ν' through head-tail damping), while Ref. 7 analyzes this effect using a two-particle model. Experimental observation of head-tail damping at the

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TRISTAN MR is analyzed in Ref. 8. A full 3-D analysis is provided in Ref. 9, and is used as a tool to measure the emittance in the TRISTAN ring.

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