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The Classical Beam-Beam Interaction for the Muon Collider: a First Look*

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Abstract

In this note we carry out some basic beam-beam studies for the muon collider in the incoherent classical approximation, taking into account the instability of the muon. This collider has a beam-beam parameter value typical of an e^+e^- collider, and a damping time that is more typical of a hadron collider. We conclude that these characteristics can coexist thanks to the short cycle time forced by the instability of the muon. We argue that classical coherent beam-beam effects will almost certainly not materialize, and neither will long tails that might spoil the beam lifetime; as a result, there is some room for upgrading the luminosity performance. We also provide some very basic constraints on the ratio β^*/σ_z and on off-center collisions arising from the hourglass effect. Finally, we attempt to prioritize work that remains to be done.

1 Introduction.

Beam-beam interaction effects can be characterized, for the purposes of practical studies, as classical or quantum, and each of these can be divided in turn into either coherent or incoherent. Obviously there are no sharp boundaries between these four areas and, at a fundamental level, they coexist. Nevertheless, depending on the parameter regime, as a practical matter one of these four dominates over the others. In addition, for multibunch colliders, there are collective effects in which all bunches in the ring "communicate" with each other via the beam-beam interaction. These multibunch affects are irrelevant for the muon collider as presently conceived and we shall not discuss them.

Classical beam-beam effects arise from the interaction of the particles in one beam with the classical electromagnetic field of the opposing beam. The fundamental dynamics is the electromagnetic deflection of the particles during the collision. Quantum beam-beam effects deal with particle annihilation and creation as described by relativistic quantum field theory.

Incoherent effects are those that are well described by the interaction of a single particle in one beam with the other beam (or by the simple superposition of such interactions), while coherent effects are those that can only be explained by the interaction of the beams with each other as whole.

A basic example of an incoherent classical effect is the blowup of the beam core (emittance blowup) as the beams collide turn after turn; as a consequence of this blowup the luminosity degrades, at least to some extent. In this case the phase space of the core particles remains essentially structureless (approximately gaussian in the case of e^+e^- machines). Another example is the development of large-amplitude tails in the particle distribution, which leads to a decrease of the beam lifetime as the particles are gradually lost to

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the machine aperture. In this case, the phase space of these large amplitude particles has a characteristic structure that is dominated by one or more resonances arising from the combined dynamics of the beam-beam force and the nonlinear magnetic fields of the machine. These two phenomena dominate the beam-beam dynamics of essentially all hadron and lepton colliders built so far. For well-tuned e^+e^- colliders with good dynamic aperture, these effects have vastly different time scales: the core blowup always happens over a few damping times, while the development of significant tails can be arranged to happen over thousands of damping times or even longer [1].

The signature for classical coherent effects is a nontrivial structure of the phase space of the core particles. This structure can arise when the tune is close to a low-order resonance. An example of this kind of effect is the flip-flop state in e^+e^- colliders: in this case, for sufficiently high bunch current, the two beams reach an equilibrium situation in which one of them is blown up while the other is not. This effect has been observed in most colliders. Other coherent states that have been predicted in simulations, and perhaps observed experimentally, are period-2 or -higher fixed points, in which the sizes of the two beams fluctuate from turn to turn in a periodic pattern. Simulations generally show that the time scale for these effects to develop is of the order of 10-20 damping times [2, 3].

An example of an incoherent quantum effect for the muon collider is the reaction $\mu^+ + \mu^- \to e^+ + e^-$ that can happen during the beam-beam collision. A muon can also interact with the collective electromagnetic field of the opposing bunch to produce e^+e^- pairs; this is a coherent quantum effect.

Quantum effects are clearly more important at high energies, hence their interest for the muon collider. These are discussed elsewhere in these proceedings. In fact, this collider is the first circular machine in which quantum effects are not a priori negligible, and deserve detailed study. In this section, however, we will show that incoherent classical effects are weak, at least for nominal parameter values, and that coherent classical effects are very unlikely to materialize. We will also provide rough criteria for the tolerances for the ratio β^*/σ_z, and for the longitudinal displacement of off-IP collisions which can arise from injection errors or from RF phasing errors.

2 Physics of the incoherent simulation.

We carry out a simulation with the code TRS [4]. This is a "strong-strong" simulation in which both beams are dynamical, and their emittances evolve according to their mutual interaction. The simulation is fully six-dimensional, and the beam-beam interaction is represented as a thick lens by dividing up the bunches into 5 "slices." We assume one bunch per beam, and a single interaction point (IP). The beams are represented by "macroparticles" (1024 per bunch in this case), and the machine lattice is assumed to be strictly linear, so that it is represented by a simple phase advance matrix. The three tunes are taken as input quantities to the simulation, and we set the chromaticity to 0. From other work, we know that the values we have chosen for the number of slices and macroparticles are adequate for the nominal muon collider specifications [5].

The beams are described at time $t = 0$ by six-dimensional gaussian distributions whose σ's are determined by the specified nominal parameters of the collider (see Table 1). We then let the bunches collide for 1000 turns, keeping track of the six-dimensional coordinates of all the macroparticles, and measure from these the beam sizes and the luminosity at every turn as they evolve according to the beam-beam dynamics. The code invokes the so-called "soft-gaussian approximation" by virtue of which, for the purposes of computing the beam-beam kick, the opposing bunch is *assumed* to have a gaussian shape in the two transverse dimensions, albeit with time-dependent σ's. This approximation is generally reliable in the absence of coherent effects, which is almost certainly the case for the muon collider, as we shall discuss below. We take into account the muon decay by simply multiplying the number of particles per bunch N in each beam by the exponential decay factor $\exp(-t/\tau)$, where τ is the Lorentz-dilated muon decay constant.

3 Beam-beam simulation.

3.1 Simulation conditions.

For the purposes of this simulation, we assume parameters as listed on Table 1 (both beams have the same parameters). In this table β^* is the common value of the horizontal and vertical beta-functions and the same

Table 1: Muon collider parameters.

C [km]	7
E [TeV]	2
N	2×10^{12}
β^* [mm]	3
ϵ_N [mm–mrad]	50
f_c [kHz]	42.86
ν_x	0.57
ν_y	0.64
ν_s	1/160
σ_z [mm]	3

is true for the normalized emittance ϵ_N. The values for the horizontal and vertical fractional tunes ν_x and ν_y were picked arbitrarily (the integral part of the tune does not enter the simulation).

With these values, the beam size at the IP is

$$\sigma^* = \sqrt{\beta^* \epsilon_N / \gamma} = 2.74 \ \mu\text{m} \tag{1}$$

where $\gamma \approx 18,900$ is the usual relativistic factor. The nominal value for the luminosity is

$$\mathcal{L} = \frac{f_c N^2}{4\pi\sigma^{*2}} = 1.82 \times 10^{35} \ \text{cm}^{-2} \ \text{s}^{-1} \tag{2}$$

which is not exactly 2×10^{35} cm^{-2} s^{-1}, as listed elsewhere in these proceedings, because it is derived from the other quantities in Table 1, which were taken to be exact.

It is worthwhile to note that the beam-beam parameter,

$$\xi = \frac{r_0 N}{4\pi\epsilon_N} = 0.046 \tag{3}$$

has a fairly typical value. In fact, beam-beam parameter values like this have been attained or exceeded in several e^+e^- colliders (here r_0 is the classical radius of the muon). In fact, it is intriguing that the values of γ and ξ are similar to those in the former PEP collider, so certain aspects of the incoherent beam-beam interaction can be expected to be similar to those in PEP.

An important parameter in colliding rings is the damping time. Assuming that the energy loss per turn in the muon collider is 4 MeV [6], the transverse damping time is

$$\tau_x = \frac{2 \ \text{TeV}}{4 \ \text{MeV}} = 0.5 \times 10^6 \ \text{turns} \tag{4}$$

which is much larger than the 1000 turns' duration of a cycle. The large difference between these time scales is crucial in explaining some beam-beam effects.

3.2 Simulation results.

Fig. 1 shows the luminosity as a function of turn number obtained under the assumption that the muon is a stable particle. One can see that it decreases by $\sim 4\%$ during the course of the 1000 turns due to the incoherent emittance blowup. This is a small fractional decrease because the beam-beam parameter is modest, and because the cycle time is so small relative to the damping time.

Fig. 2 also shows the luminosity taking into account the finite muon lifetime. As expected from the previous result, the curve is essentially determined by the exponential decay factor of the muons.

Fig. 3 shows the luminosity *vs.* turn number for three values of the number of particles per bunch N (the emittance was kept fixed at the value given in Table 1). For each value of N we carried out the simulation for three random number seeds; thus the spread in the curves for each case gives an idea of the statistical errors of the calculation. The bottom curves, corresponding to the nominal value of 2×10^{12}, are the same as in Fig. 2. The middle curves, for $N = 4 \times 10^{12}$, still behave quite nominally. However, it is clear that the curves for $N = 6 \times 10^{12}$ decay faster than exponentially due to substantial emittance blowup. In addition, when we included a 10σ physical aperture in the simulation, we observed that there were no particle losses for the first two cases, but there was a $\sim 2\%$ integrated beam loss for $N = 6 \times 10^{12}$. Although this is a small fraction of particles, it is reasonable to interpret this as a symptom that the beam-beam strength is being pushed beyond a prudent limit, and the results of this simulation cannot be taken as a reliable guide. When this kind of behavior is seen, it is likely that other detrimental effects, not included in this simulation, will become important and will lead to even more unfavorable behavior. We conclude from this calculation that the incoherent beam-beam effect is weak for the nominal current and that there is some room for upgrading the luminosity by increasing the bunch current by a factor of ~ 2 but not more than this.

4 Other classical beam-beam issues.

4.1 Coherent effects.

Classical coherent effects significantly distort the phase space of the beam core away from the gaussian shape. This distortion may be static or time dependent, and leads to luminosity degradation; thus, despite the theoretical interest of these effects, in practice one wants to identify the conditions under which they appear in order to avoid them.

Simulation studies for e^+e^- machines [2] show that these effects materialize for beam-beam parameter values $\gtrsim 0.05$ and for isolated values of the fractional tune where certain low-order resonances dominate the dynamics. More importantly, these results also show that coherent effects take a long time to develop, on the order of 10 damping times or more, simply because it takes a long time for the particles to redistribute in phase space in order to give rise to a clear structure. At the beginning of the simulation (the time scale being set by the damping time), there is no hint of structure, and the phase space distribution is essentially gaussian. Furthermore, these results are obtained in the zero-bunch-length approximation, and there are indications [7] that a nonzero bunch length strongly suppresses coherent effects. Although more research is needed, and experimental work under controlled conditions needs to be carried out to confirm the simulation results, we can safely conclude from presently available information that these effects are unlikely to appear in the muon collider.

4.2 Beam tails and beam-beam lifetime.

While the beam core determines the luminosity of a collider, the beam tails determine the beam lifetime. The beam core, typically understood to be the phase space region within $\sim 3\sigma$ of the center, is not very sensitive to nonlinear machine resonances because the lattice magnetic fields are typically quite linear in this region. On the other hand, the beam tail extends out to sufficiently large amplitudes so that its dynamics is sensitive to an interplay [8] of beam-beam and lattice nonlinearities (beam-gas scattering can also come into play in subtle ways, although typically it has a clearer effect on the beam core).

There has been much recent progress in understanding and properly simulating the beam tails in e^+e^- colliders. These new tools make use of a clever algorithm by which the brute-force tracking is "accelerated" by 2-3 orders of magnitude in order to determine the particle density and flux at large amplitudes (up to $\sim 20\sigma$ or so) [9, 10]. From the particle flux one can then determine the lifetime, given the physical aperture. For the purposes of this article, one can roughly summarize the conclusions as follows: for a lattice with reasonable dynamic aperture (meaning 10σ or more), and for reasonable values of the beam-beam parameter (meaning 0.05 or less), it is not difficult to find working points for which the beam-beam lifetime is of the order of $10^7 - 10^9$ turns (however, the lifetime can degrade by several orders of magnitude by relatively small changes in these parameters). In any case, the instability of the muon will almost certainly dominate the beam lifetime, so at least from the perspective of luminosity lifetime, the beam tails will not be important.

Thus the beam tails might be much more important for other reasons such as background and radiation. The important thing, therefore, is to specify the maximum acceptable number of muons that can hit the vacuum chamber during the 1000 turns of a cycle. Such a criterion is closely related to that of the dynamic aperture. In the above-mentioned e^+e^- simulations, the damping time, typically of order $10^3 - 10^4$ turns, also plays an important role. The muon collider, as mentioned earlier, has negligible damping, so in this respect it is akin to proton colliders. It seems therefore that the tracking tools used to determine the dynamic aperture of such machines are the right ones for this case, provided they are augmented to include a beam-beam element. Such a code development should be relatively simple, although the analysis will likely involve many iterations.

4.3 Hourglass effect for centered collisions.

Because of the geometrical divergence of the beams at the IP, the luminosity is actually smaller than the nominal value given by Eq. (2), which represents the limiting value as $\sigma_z \to 0$. As σ_z grows at fixed β^*, the luminosity decreases due to this "hourglass effect." Neglecting all dynamical effects, this purely geometrical reduction factor is given, for symmetric round gaussian beams, by the formula [11]

$$\frac{\mathcal{L}(\sigma_z)}{\mathcal{L}(0)} = \int_{-\infty}^{\infty} \frac{dt}{\sqrt{\pi}} \frac{e^{-t^2}}{1+(t/t_x)^2} = \sqrt{\pi}\, t_x\, e^{t_x^2}\, \mathrm{erfc}(t_x) \tag{5}$$

where $t_x \equiv \beta^*/\sigma_z$. For the muon collider, t_x has been chosen to be unity; in order to get an idea of the sensitivity to this parameter, we show in Fig. 4 the reduction factor given by the above formula. One can see that the luminosity degrades quickly as σ_z increases.

4.4 Hourglass effect for longitudinally-displaced collisions.

By virtue of the hourglass effect, the luminosity also degrades if the bunches collide at a point away from the optical IP. If the central collision is longitudinally displaced by a distance s_c from the IP (but the bunches still collide transversely head-on), the luminosity reduction factor is given by [11]

$$\frac{\mathcal{L}(s_c,\sigma_z)}{\mathcal{L}(0,0)} = \int_{-\infty}^{\infty} \frac{dt}{\sqrt{\pi}} \frac{e^{-(t-t_z)^2}}{1+(t/t_x)^2} \tag{6}$$

where $t_z \equiv s_c/\sigma_z$ and t_x is the same as above. In Fig. 5 we show the luminosity reduction factor as a function of t_z (please note that in this figure we have normalized the reduction factor to $\mathcal{L}(0,\sigma_z)$ and *not* to $\mathcal{L}(0,0)$ as in Eq. (6)). One can see that the luminosity degrades quickly when the collision point is farther away than $\sim 1\sigma_z$ from the optical IP. This gives an idea of the phase errors that can be tolerated.

5 Conclusions.

From the perspective of the classical beam-beam dynamics, the four key features that distinguish the muon collider are:

1. A relatively modest beam-beam parameter, $\xi = 0.046$.
2. A short cycle of 1000 turns.
3. A long damping time, $\tau_x = 0.5 \times 10^6$ turns.
4. Unstable muons.

The first feature is shared with many e^+e^- colliders; the second makes this collider not too different from single-pass colliders; the third one makes it resemble a hadron collider; and the fourth, of course, is unique to this machine. It is fair to say that one can understand all features of the classical beam-beam interaction from the interplay of these four characteristics.

We have shown by means of beam-beam simulations that the classical incoherent beam-beam effect is quite weak for the muon collider in its present design. From this perspective, there is room for upgrading the luminosity, if necessary, by increasing the bunch current by a factor of 2 or so but not more than this.

We have argued that coherent classical beam-beam effects are very unlikely to materialize simply because a 1000-turn cycle is too short.

We have also argued that beam tails are unlikely to affect the luminosity lifetime. Undoubtedly there will be a certain number of large-amplitude muons that will hit the chamber, and it seems important to establish this number. This issue is closely related to the determination of the dynamic aperture, and single-particle tracking tools used for hadron colliders, duly augmented to include the beam-beam interaction, seem appropriate to address this issue.

From purely geometrical considerations, we have provided a rough estimate (probably a lower bound) of the sensitivity of the luminosity to the ratio β^*/σ_z and to the longitudinal displacement of the collision point from the IP. These estimates yield fairly standard results: one should not choose the ratio β^*/σ_z below ~ 1 or so, and one should not allow collisions to be displaced from the optical IP by more than $\sim 1\sigma_z$ in either direction.

Much work remains to be done to firm up the limits imposed by the beam-beam interaction. Here is a brief suggested list, roughly in order of priority:

1. Develop a dynamic aperture tool by augmenting a single-particle tracking code to include a "beam-beam lens."
2. Track specific lattices, including all nonlinearities, and estimate from the results the number of muons that hit the vacuum chamber during 1000 turns; iterate this process to determine tolerances on machine nonlinearities.
3. Establish the sensitivity of the beam-beam interaction to longitudinal and transverse alignment errors and jitter.

6 Acknowledgments.

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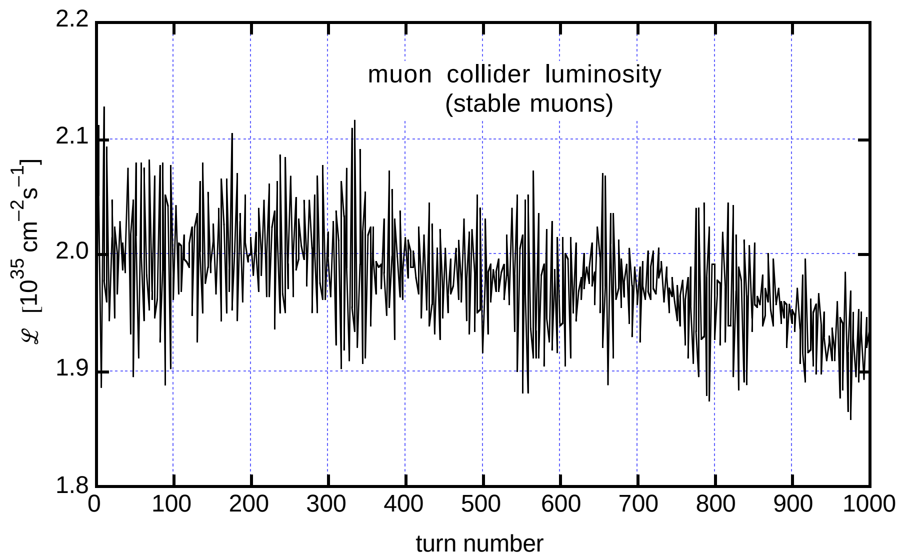

Figure 1: Luminosity as a function of turn number assuming that the muons are stable particles. The luminosity degrades by $\sim 4\%$ over 1000 turns due to incoherent emittance blowup.

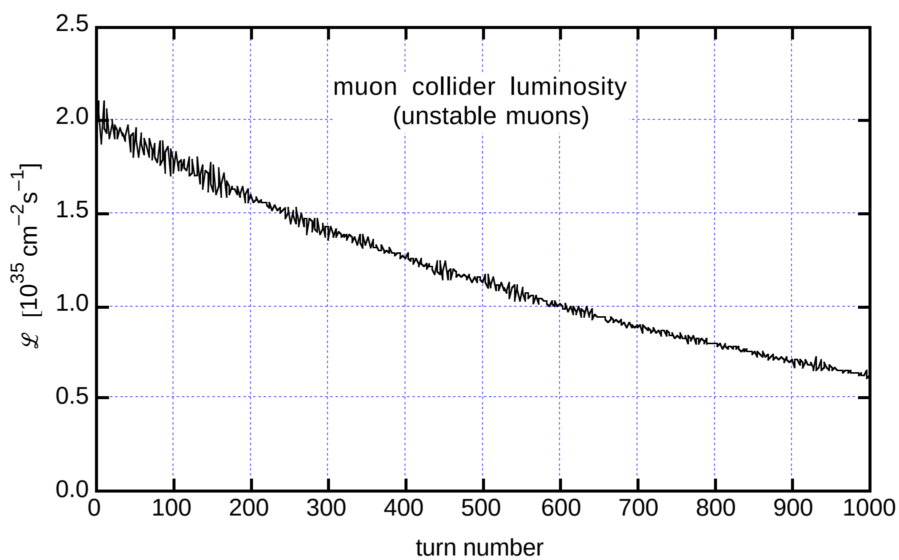

Figure 2: Luminosity as a function of turn number, taking into account the finite muon lifetime. The curve follows closely the expected exponential decay dependence.

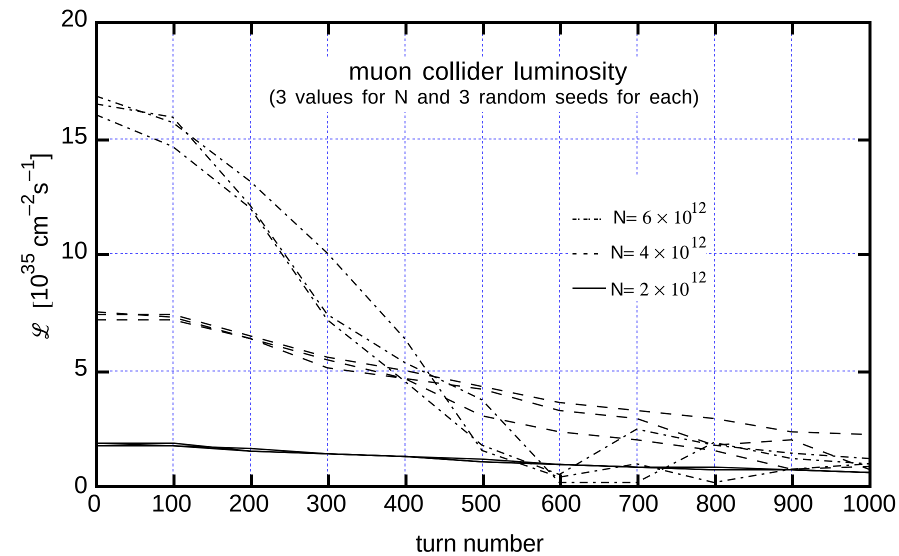

Figure 3: Luminosity as a function of turn number for three different values of the number of particles per bunch N. For each case we show three runs, each corresponding to a different random number seed; the spread of the curves for each case gives an idea of the statistical accuracy of the calculation.

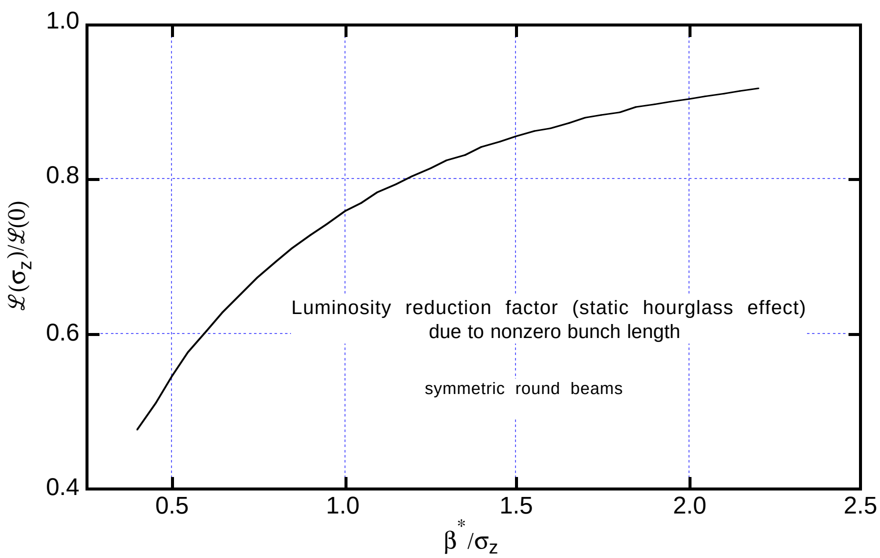

Figure 4: The hourglass luminosity reduction factor as a function of the ratio β^*/σ_z. The normalization is $\mathcal{L}(\sigma_z = 0)$.

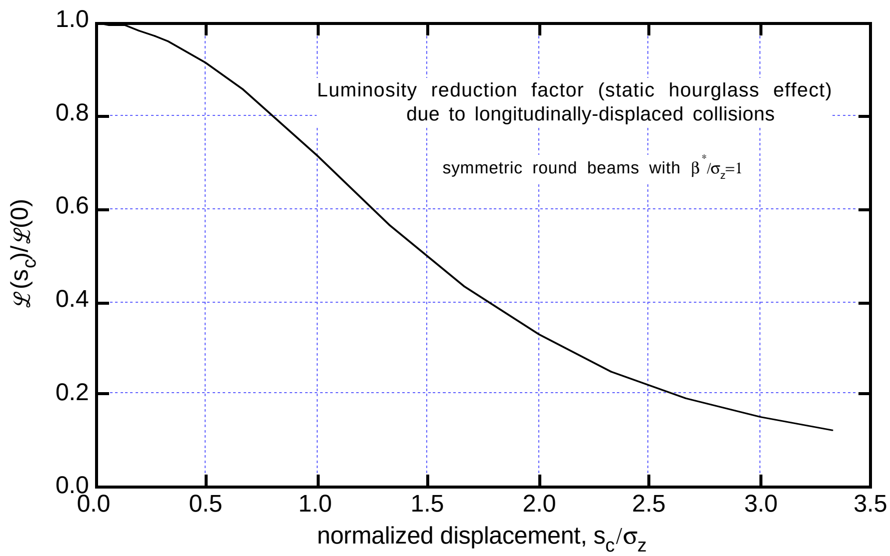

Figure 5: The hourglass luminosity reduction factor when the collisions are longitudinally displaced from the IP by a distance s_c, plotted as a function of s_c/σ_z. Note that the normalization is $\mathcal{L}(s_c = 0, \sigma_z = 3\ \mathrm{mm})$, *i.e.* the nominal value.