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Beam-Beam Tune Shift and Dynamical Beta Function in PEP-II†

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Abstract

We present the calculation of the beam-beam tune shift and dynamical beta function for PEP-II as a function of the fractional tune and the beam separation at the parasitic collision (PC) points. We do the calculation both for "typical" and for "pacman" bunches taking into account all the PCs.

1 INTRODUCTION

If the beam-beam interaction is relatively weak, as is the case for most colliders, one can assess, in linear approximation, its most basic constraint on the choice of the working point. This constraint is absolutely necessary, although far from sufficient, for acceptable luminosity performance. Obviously this approximation is insensitive to all synchro-betatron resonances, and to all betatron resonances except those near the integer and half-integer tunes.

There are three well-known consequences that follow from the linear approximation: (1) stopbands near integer and half-integer tunes appear; (2) the tune shift produced by the beam-beam collision is significantly different from the beam-beam parameter near the edges of the stopband; and (3) the beta function at the IP is different from its nominally-specified value (this is the so-called "dynamical beta function" effect). We compute here the edges of the stopbands, the beam-beam tune shifts and the dynamical beta functions at the IP for the specific case of PEP-II, as a function of tune.

The PEP-II design [1] calls for head-on collisions with magnetic separation in the horizontal plane. As a result, there are four PCs on either side of the IP. If the beam were uniformly populated, the tune shift and dynamical beta function would be the same for all bunches. However, the existence of an ion-clearing gap implies that those bunches at the head and the tail of the train (dubbed "pacman" bunches) do not experience all PCs. For this reason, these bunches have a different tune shift and dynamical beta function from the bunches away from the ends of the train (dubbed "typical" bunches). In our calculation we take into account all the PCs, and we present results both for typical and for pacman bunches. This article summarizes Ref. [2].

2 CALCULATION IN LINEAR THEORY

Each beam-beam collision, whether it is head-on or long-range, is characterized in lowest order by a beam-beam parameter ξ which measures its strength as experienced by the particle at the center of the bunch. In the small-amplitude approximation it is described by the kick matrix

$$K(n) \equiv \begin{pmatrix} 1 & 0 \\ -4\pi\xi_n/\beta_n & 1 \end{pmatrix} \tag{1}$$

where β_n is the lattice beta-function at the collision point n.

We assume that the lattice is linear and that there is no x-y coupling, so that we can treat the horizontal and vertical phase spaces separately in the two rings. We label the parasitic collisions $n = 1, \cdots, 4$ or $n = -4, \cdots, -1$ as shown in Fig. 1, and $n = 0$ is the main collision at the IP. With this convention, the one-turn map for a particle corresponding to a surface of section immediately before the IP is given by

$$\begin{aligned} M'(0) = M(0,-1)K(-1)\cdots K(-4) \\ \times M(-4,4)K(4)\cdots K(1)M(1,0)K(0) \end{aligned} \tag{2}$$

where $M(n,m)$ is the linear transport matrix [3] from point m to point n.

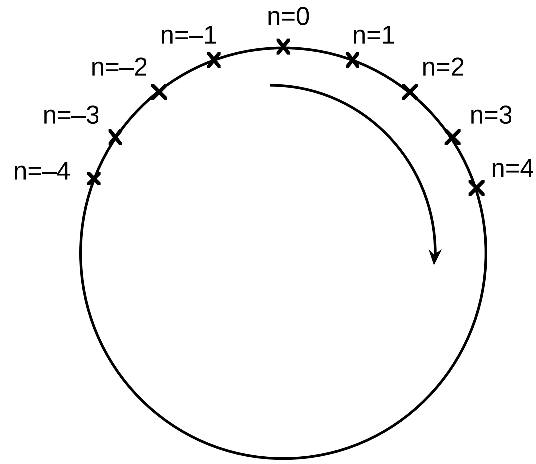

Fig. 1: Sketch of the beam-beam collisions around the ring. n=0 represents the main collision at the IP. The others collisions are parasitic. The beam moves in the direction indicated by the arrow.

The beam-beam tune shift $\Delta\nu$ and the dynamical beta function β' at the IP are extracted from the usual formulas

$$\cos(2\pi(\nu+\Delta\nu)) = \mathrm{tr}\, M'(0)/2 \tag{3}$$

$$\beta' = \frac{M'(0)_{12}}{\sin(2\pi(\nu+\Delta\nu))} \tag{4}$$

where ν is the unperturbed, or "bare lattice," tune. For a range of values of the tune, defined by $\nu_- \le \nu \le \nu_+$, the right-hand side of Eq. (3) is larger than 1 in absolute value, and hence a stopband appears. $\Delta\nu$ reaches a finite limit at both edges of the stopband, while β' is infinite at ν_- and zero at ν_+. For a single kick of strength ξ_0, $\nu_+ = p/2$ (exactly), and $\nu_- = p/2 - 2\xi_0 + O(\xi_0^2)$, where p is an arbitrary integer.

Taking the PCs into account [2] we obtain, to lowest-

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order[1] in the ξ's,

$$\Delta\nu = \sum_{n=-4}^{4} \xi_n + \cdots \qquad (5)$$

$$\frac{\beta'}{\beta} = 1 - \frac{2\pi}{\sin(2\pi\nu)} \sum_{n=-4}^{4} \xi_n \cos\left(2\Delta\phi_n - 2\pi\nu\right) + \cdots \qquad (6)$$

where $\Delta\phi_n$ is the phase advances, modulo $2\pi\nu$, of the collision points relative to the IP. If the optics of the IR is symmetrical about the IP, as is the case in PEP-II, then the stopband edges $\nu_\pm$ and width $\delta\nu \equiv \nu_+ - \nu_-$ are given by [2]

$$\begin{aligned} \nu_+ &= p/2 - 4\sum_{n\geq 1} \xi_n \sin^2 \Delta\phi_n + \cdots \\ \nu_- &= p/2 - 2\xi_0 - 4\sum_{n\geq 1} \xi_n \cos^2 \Delta\phi_n + \cdots \end{aligned} \qquad (7)$$

$$\delta\nu = 2\xi_0 + 4\sum_{n\geq 1} \xi_n \cos 2\Delta\phi_n + \cdots \qquad (8)$$

3 APPLICATION TO PEP-II

The input to the calculation is the set of ξ's and $\Delta\phi$'s for all collisions. The lattice functions at the PCs do not enter if the calculation is carried out in locally-normalized coordinates [2]. The ξ's are computed from the usual formulas; numerical values for the high-energy beam (HEB) and the low-energy beam (LEB) are listed in Ref. [1] (the lattices are different for the two rings). The key parameter is ξ_0, the nominal beam-beam parameter at the IP; it has the value 0.03 for both beams in both planes. At the PCs, $\xi_n \propto d_n^{-2}$; the strongest PC is the first one, for which the beam separation is the smallest.

3.1 Results for typical bunches

Figure 2 shows the tune shift for a typical bunch plotted vs. the bare lattice tune. One sees that the vertical tune shifts, particularly that of the LEB, are clearly higher than the nominal beam-beam parameter value of 0.03. The horizontal tune shift becomes small just above the integer (or half-integer), and the vertical tune shift becomes small just below the half-integer (or integer).

The location and width of the horizontal stopbands can be well understood from Eqs. (7-8). The downshifts of the vertical stopbands are accounted for by the fact that the vertical ξ's are >0. It is interesting to note that the PCs tend to make the stopbands *narrower* than if they were due to the IP alone. This is particularly true for the vertical stopbands, for which this narrowing is explained by noting that the $\Delta\phi$'s are all very close to $\pi/2$, hence $\cos 2\Delta\phi_n \approx -1$ in Eq. (8). The remarkable (but approximate) coincidence of the four lower edges ν_- of the stopbands is due to a numerical accident involving the values of the phase advances and the beam-beam parameters of the PCs.

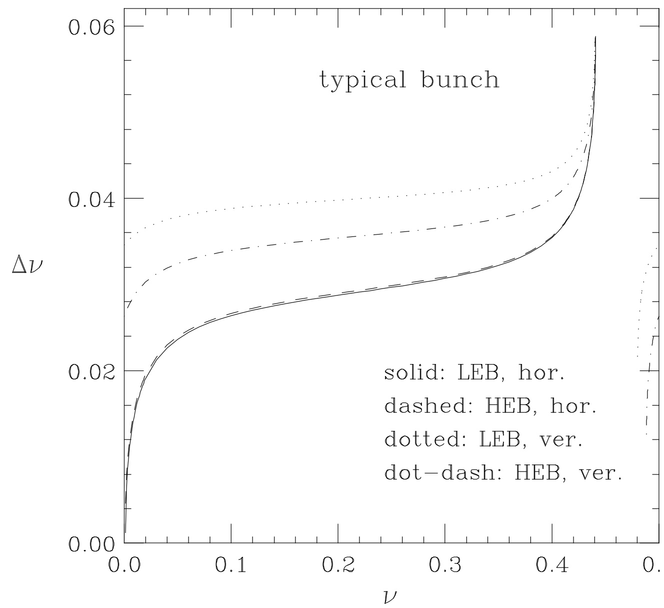

Fig. 2: The horizontal and vertical beam-beam tune shift for a typical bunch as a function of the corresponding tune for nominal PEP-II parameters. The figure is periodic in ν with a period of 0.5.

Figure 3 shows the dynamical beta functions, normalized to their nominal values, plotted vs. tune. One can see that the dynamical beta functions are smaller than their nominal counterparts for tunes ñ0.25. This is qualitatively explained by the dominance of the IP term (n=0) in Eq. (6), since $\cot 2\pi\nu$ is >0 for ν<0.25. The difference between the four curves in Fig. 3 is due to the PCs: if the PCs were ignored, the four curves would overlap.

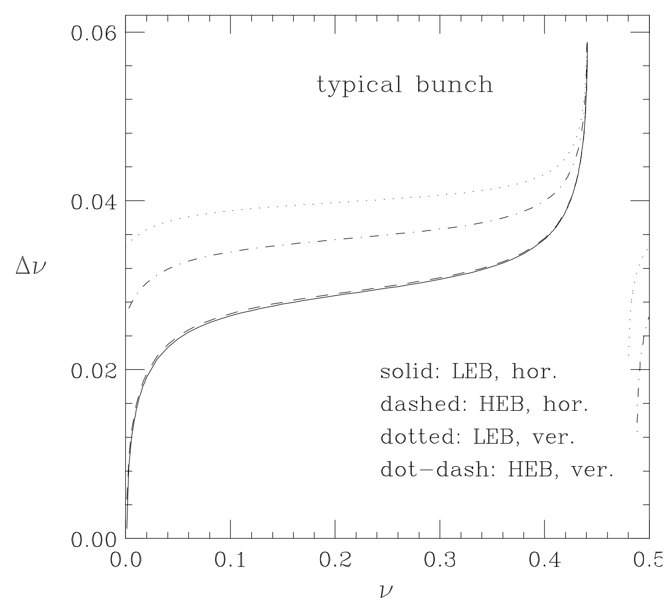

Fig. 3: The horizontal and vertical normalized dynamical beta function for a typical bunch as a function of the corresponding tune for nominal PEP-II parameters. The figure is periodic in ν with a period of 0.5.

[1] Eq. (4.49) in Ref. [3] has two sign errors which, unfortunately, have propagated through much of the literature. The equations leading up to (4.49) are correct, but there is a trigonometric error at the very last step of the derivation. Our Eq. (6) is the correct result for discrete kicks.

3.2 Results for pacman bunches.

Figure 4 shows the beam-beam tune shifts for the first pacman bunch, i.e., the bunch at the head of the train. This bunch experiences the main collision at the IP plus the PCs at one side of the IP only. By symmetry, the results for the last bunch at the tail of the beam are identical to those for the head bunch.

The beam-beam tune shifts for the other pacman bunches are in between those for the first pacman bunch and those for a typical bunch. By comparing Figs. 2 and 4, one can see that there is almost no difference for the horizontal tune shifts, since for these the PCs are quite negligible. For the vertical tune shifts, the effect of the PCs for the first pacman bunch are, roughly speaking, about half as strong as for a typical bunch, hence the values for the tune shifts are about half way in between the horizontal values and those for a typical bunch. By the same reasoning, the horizontal normalized dynamical beta functions for the head bunch (not shown) are almost exactly the same as those for a typical bunch, while the vertical normalized dynamical beta functions are somewhere in between the horizontal values and those for a typical bunch.

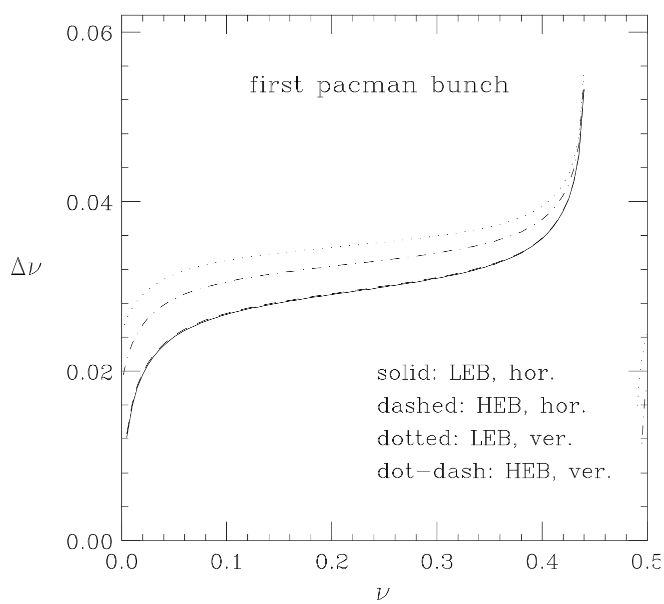

Fig. 4: The horizontal and vertical beam-beam tune shift for the first pacman bunch, as a function of the corresponding tune for nominal PEP-II parameters. The figure is periodic in ν with a period of 0.5.

3.3 Results when only the first PCs are considered.

The effect on the beam dynamics of the beam separation d_1 at the first PC has been extensively studied by simulation [1,4]. Figure 5 shows the beam-beam tune shifts of a typical bunch plotted vs. d_1. In this calculation all PCs beyond the first have been neglected (the first PC is significantly stronger than the others [1]), and d_1 is taken as a free parameter. One can see that the vertical tune shift, particularly that of the LEB, becomes large quickly as the beam separation decreases from its nominal value.

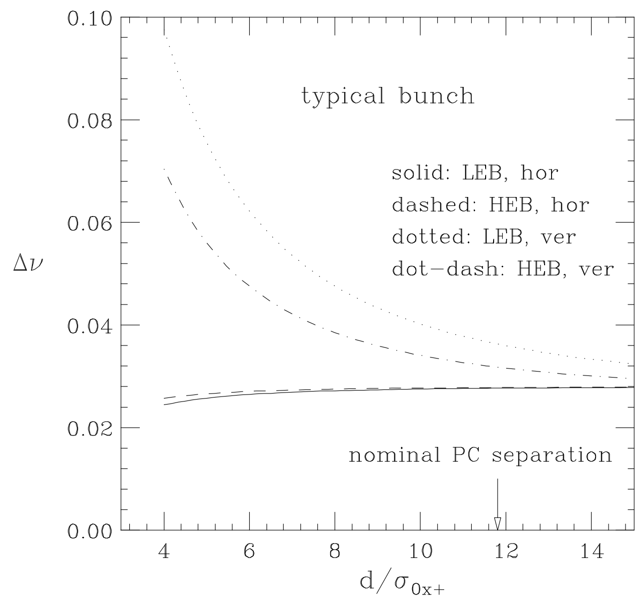

Fig 5: The beam-beam tune shifts of a typical bunch as a function of the beam separation at the first PC. The beam separation is normalized to the local nominal horizontal beam size of the LEB. The fractional tunes are fixed at (ν_x, ν_y)=(0.64, 0.57) for both beams, and all other parameters have their nominal PEP-II values. The arrow indicates the nominal separation, as specified in the CDR [1].

4 CONCLUSIONS

We conclude that: (1) It is advantageous to choose a working point just above the integer or the half-integer because the dynamical beta function is smaller than nominal and the beam-beam tune shift is smaller than the beam-beam parameter. (2) The vertical beam-beam tune shifts and dynamical beta functions, especially those of the LEB, are much more sensitive than the horizontal ones to the beam separation at the PC: for small enough separation, both the tune shift and the dynamical beta function become large, undoing the favorable effect of the choice of the working point. (3) The tune shift and the dynamical beta function as a function of the beam separation at the first PC for a fixed working point shows that the vertical quantities are quite sensitive to d_1, a result which correlates well with the beam blowup observed in multiparticle simulations [1,4].

5 REFERENCES

[1] “PEP-II: An Asymmetric B Factory Conceptual Design Report,” June 1993, LBL-PUB-5379/SLAC-418/CALT-68-1869/UCRL-ID-114055/UC-IIRPA-93-01.

[2] M. A. Furman, “Dynamical Beta Function and Beam-Beam Tune Shift for PEP-II,” PEP-II/AP Note 4-93/ESG Tech Note-232, January, 1994.

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[4] M. A. Furman and J. R. Eden, “Beam-Beam Effects for the PEP-II B Factory,” Proc. Part. Accel. Conf., Washington, DC, May 17–20, 1993, p. 3485 and references therein.